Exercises practical 1

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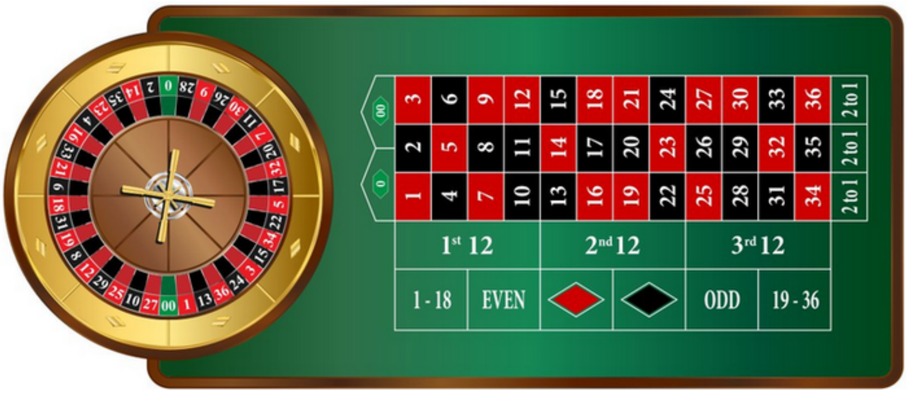
SEM 2: Structural Equation Modeling

The *American Roulette* casino game is slightly different from the regular (French) roulette game described in the lecture, in that an extra "00" field is added, leading to a total of 38 spaces the ball can fall on:



The following table gives an overview of some bets you can place:

bet type	payout	probability	expected value	standard deviation
single number (e.g., number 17)	35 to 1	1/38	...	...
single row (e.g., 4, 5, and 6)	11 to 1	...	...	...
single column (e.g., all values in first column, except for 0)	2 to 1	...	...	...
all red or black numbers	1 to 1	...	...	...
top line bet (bet on numbers 00, 0, 1, 2, and 3)	6 to 1	...	...	...

The payout of 35 to 1 indicates that betting € 10 on the right number leads to a profit of € 350, and betting € 10 on the wrong number leads to losing the 10 Euro.

Exercise 1 Fill in the missing cells in the table above, assuming a bet of € 10. ■

In Dungeons & Dragons (DnD; 3rd edition), I play a half-orc Barbarian that is not very smart, but can use the largest weapons possible. Three such weapons would be the following:


(a) A magical greatsword with 2d6 + 1 damage

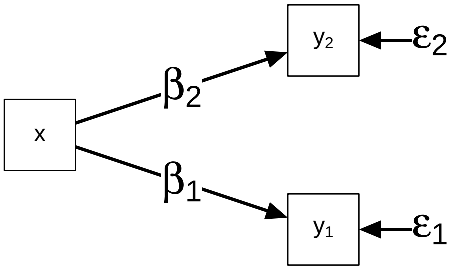

(b) A magical flail with 1d10 + 2 damage


(c) A magical greataxe with 1d12 + 1 damage

In DnD, the notation "XdY + Z" stands for "throw X dices with Y sides, and add Z". This makes use of some special dices that have more sides (e.g., a 10-sided and a 12-sided die). The final value is the damage a weapon does.

Exercise 2 Calculate the expected value and standard deviation of each of these weapons. Which should I prefer? ■

The following path diagram (ignoring the variance of the exogenous variable x):

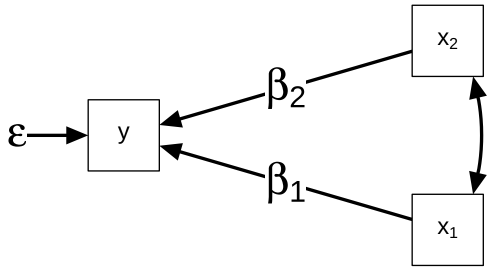


encodes the following structural equations (ignoring intercepts):

$$y_{i1} = \beta_1 x_i + \varepsilon_{i1}$$
$$y_{i2} = \beta_2 x_i + \varepsilon_{i2}$$

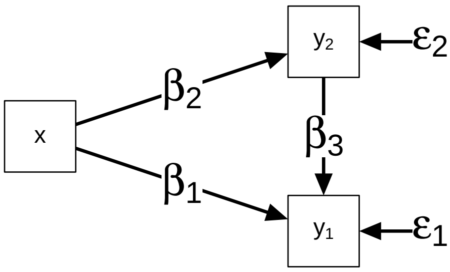
Exercise 3
Derive $\text{Var}(y_1)$ and $\text{Cov}(y_1, y_2)$.

Now consider the following path diagram:



Exercise 4
Write down the structural equation for y and derive $\text{Var}(y)$.

Finally, consider:



Exercise 5
Write down the structural equations for y_1 and y_2 and derive $\text{Var}(y_1)$.