Exercises practical 1

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SEM 2: Structural Equation Modeling
The *American Roulette* casino game is slightly different from the regular (French) roulette game described in the lecture, in that an extra “00” field is added, leading to a total of 38 spaces the ball can fall on:

The following table gives an overview of some bets you can place:

<table>
<thead>
<tr>
<th>Bet Type</th>
<th>Payout</th>
<th>Probability</th>
<th>Expected Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single number (e.g., number 17)</td>
<td>35 to 1</td>
<td>1/38</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Single row (e.g., 4, 5, and 6)</td>
<td>11 to 1</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Single column (e.g., all values in first column, except for 0)</td>
<td>2 to 1</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>All red or black numbers</td>
<td>1 to 1</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Top line bet (bet on numbers 00, 0, 1, 2, and 3)</td>
<td>6 to 1</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

The payout of 35 to 1 indicates that betting €10 on the right number leads to a profit of €350, and betting €10 on the wrong number leads to losing the 10 Euro.

**Exercise 1** Fill in the missing cells in the table above, assuming a bet of €10.

In *Dungeons & Dragons* (DnD; 3rd edition), I play a half-orc Barbarian that is not very smart, but can use the largest weapons possible. Three such weapons would be the following:

(a) A magical greatsword with 2d6 + 1 damage

(b) A magical flail with 1d10 + 2 damage

(c) A magical greataxe with 1d12 + 1 damage

In DnD, the notation “XdY + Z” stands for “throw X dices with Y sides, and add Z”. This makes use of some special dice that have more sides (e.g., a 10-sided and a 12-sided die). The final value is the damage a weapon does.

**Exercise 2** Calculate the expected value and standard deviation of each of these weapons. Which should I prefer?
The following path diagram (ignoring the variance of the exogenous variable $x$):

The following path diagram encodes the following structural equations (ignoring intercepts):

\begin{align*}
y_{i1} &= \beta_1 x_i + \epsilon_{i1} \\
y_{i2} &= \beta_2 x_i + \epsilon_{i2}
\end{align*}

**Exercise 3**
Derive $\text{Var}(y_1)$ and $\text{Cov}(y_1, y_2)$.

Now consider the following path diagram:

**Exercise 4**
Write down the structural equation for $y$ and derive $\text{Var}(y)$.

Finally, consider:

**Exercise 5**
Write down the structural equations for $y_1$ and $y_2$ and derive $\text{Var}(y_1)$. 