

SEM 2: Structural Equation Modeling

Week 3 - Partial covariance

Sacha Epskamp

If multivariate normality holds, then the Schur complement shows that **any** partial covariance can be expressed solely in terms of variances and covariances:

$$\text{Cov}(\mathbf{y}_i, \mathbf{y}_j \mid \mathbf{x}) = \text{Cov}(\mathbf{y}_i, \mathbf{y}_j) - \text{Cov}(\mathbf{y}_i, \mathbf{x}) \text{Var}(\mathbf{x})^{-1} \text{Cov}(\mathbf{x}, \mathbf{y}_j)$$

in the multivariate case and:

$$\text{Cov}(y_i, y_j \mid x) = \text{Cov}(y_i, y_j) - \frac{\text{Cov}(y_i, x) \text{Cov}(x, y_j)}{\text{Var}(x)}$$

in the univariate case.

If multivariate normality holds, then the Schur complement shows that **any** partial covariance can be expressed solely in terms of variances and covariances:

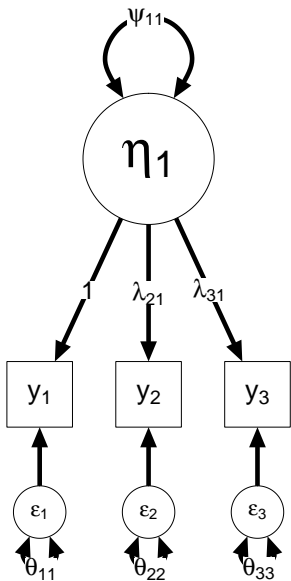
$$\text{Cov}(\mathbf{y}_i, \mathbf{y}_j \mid \mathbf{x}) = \text{Cov}(\mathbf{y}_i, \mathbf{y}_j) - \text{Cov}(\mathbf{y}_i, \mathbf{x}) \text{Var}(\mathbf{x})^{-1} \text{Cov}(\mathbf{x}, \mathbf{y}_j)$$

in the multivariate case and:

$$\text{Cov}(y_i, y_j \mid x) = \text{Cov}(y_i, y_j) - \frac{\text{Cov}(y_i, x) \text{Cov}(x, y_j)}{\text{Var}(x)}$$

in the univariate case.

- ▶ All conditional relationships can be expressed in terms of Σ !
- ▶ If we know Σ , we know everything we can about the relationships between variables.
- ▶ Fitting a SEM model equals simultaneously testing all conditional independence relationships implied by the model!

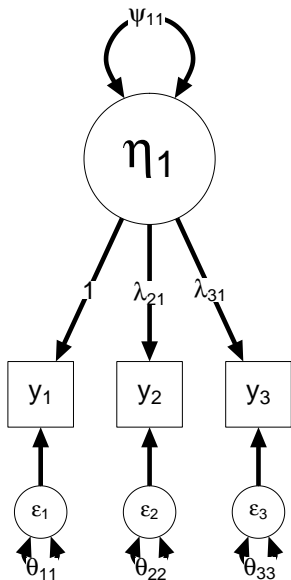


$$\text{Cov}(y_1, y_2) = \psi_{11} \lambda_{21}$$

$$\text{Cov}(y_1, \eta_1) = \psi_{11}$$

$$\text{Cov}(y_2, \eta_1) = \lambda_{21} \psi_{11}$$

$$\text{Var}(\eta_1) = \psi_{11}$$



$$\text{Cov}(y_1, y_2) = \psi_{11} \lambda_{21}$$

$$\text{Cov}(y_1, \eta_1) = \psi_{11}$$

$$\text{Cov}(y_2, \eta_1) = \lambda_{21} \psi_{11}$$

$$\text{Var}(\eta_1) = \psi_{11}$$

Now using the Schur compliment:

$$\text{Cov}(y_1, y_2 | \eta_1) = \text{Cov}(y_1, y_2) -$$

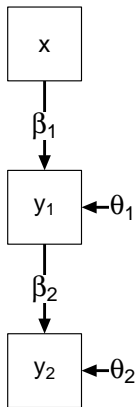
$$\text{Cov}(y_1, \eta_1) \text{Var}(\eta_1)^{-1} \text{Cov}(\eta_1, y_2)$$

$$= \psi_{11} \lambda_{21} - \psi_{11} \psi_{11}^{-1} \lambda_{21} \psi_{11}$$

$$= \psi_{11} \lambda_{21} - \psi_{11} \psi_{11}^{-1} \lambda_{21} \psi_{11}$$

$$= \psi_{11} \lambda_{21} - \lambda_{21} \psi_{11}$$

$$= 0$$

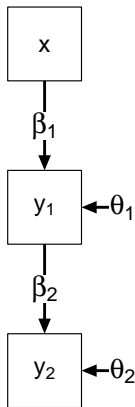


$$\text{Cov}(x, y_2) = \beta_2 \beta_1 \text{Var}(x)$$

$$\text{Cov}(x, y_1) = \beta_1 \text{Var}(x)$$

$$\text{Cov}(y_1, y_2) = \beta_2 \beta_1^2 \text{Var}(x) + \beta_2 \theta_1$$

$$\text{Var}(y_1) = \beta_1^2 \text{Var}(x) + \theta_1$$



$$\text{Cov}(x, y_2) = \beta_2 \beta_1 \text{Var}(x)$$

$$\text{Cov}(x, y_1) = \beta_1 \text{Var}(x)$$

$$\text{Cov}(y_1, y_2) = \beta_2 \beta_1^2 \text{Var}(x) + \beta_2 \theta_1$$

$$\text{Var}(y_1) = \beta_1^2 \text{Var}(x) + \theta_1$$

Now using the Schur compliment:

$$\text{Cov}(x, y_2 | y_1) = \text{Cov}(x, y_2) -$$

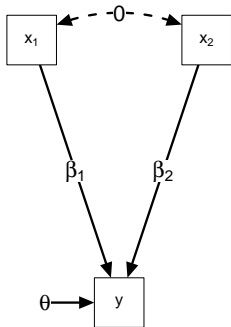
$$\text{Cov}(x, y_1) \text{Var}(y_1)^{-1} \text{Cov}(y_2, y_1)$$

$$= \beta_2 \beta_1 \text{Var}(x) - \frac{\beta_1 \text{Var}(x) (\beta_2 \beta_1^2 \text{Var}(x) + \beta_2 \theta_1)}{\beta_1^2 \text{Var}(x) + \theta_1}$$

$$= \beta_2 \beta_1 \text{Var}(x) - \beta_2 \beta_1 \text{Var}(x) \frac{\beta_1^2 \text{Var}(x) + \theta_1}{\beta_1^2 \text{Var}(x) + \theta_1}$$

$$= \beta_2 \beta_1 \text{Var}(x) - \beta_2 \beta_1 \text{Var}(x)$$

$$= 0$$

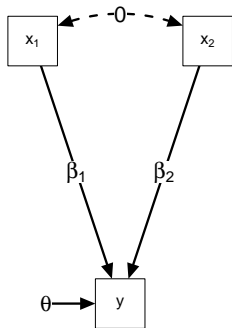


$$\text{Cov}(x_1, x_2) = 0$$

$$\text{Cov}(x_1, y) = \beta_1 \text{Var}(x_1)$$

$$\text{Cov}(x_2, y) = \beta_2 \text{Var}(x_2)$$

$$\text{Var}(y) = \theta + \beta_1^2 \text{Var}(x_1) + \beta_2^2 \text{Var}(x_2)$$



$$\text{Cov}(x_1, x_2) = 0$$

$$\text{Cov}(x_1, y) = \beta_1 \text{Var}(x_1)$$

$$\text{Cov}(x_2, y) = \beta_2 \text{Var}(x_2)$$

$$\text{Var}(y) = \theta + \beta_1^2 \text{Var}(x_1) + \beta_2^2 \text{Var}(x_2)$$

Now using the Schur complement:

$$\begin{aligned} \text{Cov}(x_1, x_2 | y) &= \text{Cov}(x_1, x_2) - \text{Cov}(x_1, y) \text{Var}(y)^{-1} \text{Cov}(x_2, y) \\ &= 0 - \frac{\beta_1 \text{Var}(x_1) \beta_2 \text{Var}(x_2)}{\theta + \beta_1^2 \text{Var}(x_1) + \beta_2^2 \text{Var}(x_2)} \\ &= \begin{cases} < 0 & \text{if } \text{sign}(\beta_1) = \text{sign}(\beta_2) \\ > 0 & \text{if } \text{sign}(\beta_1) \neq \text{sign}(\beta_2) \end{cases} \end{aligned}$$

Psychological Networks in Clinical Populations: A tutorial on the consequences of Berkson's Bias

AUTHORS

Jill de Rooi, Eiko Fried, Sacha Epskamp

CREATED ON

January 19, 2019

LAST EDITED

January 19, 2019

SUPPLEMENTAL MATERIALS[osf.io/5t8zw](https://doi.org/10.21203/psyarxiv.5t8zw)

Page: 1 of 21 Automatic Zoom

Running head: **Berkson's Bias in Psychological Networks** 1

**Psychological Networks in Clinical Populations:
A tutorial on the consequences of Berkson's Bias**

Jill De Rooi¹*
Eiko I. Fried²
Sacha Epskamp¹

¹University of Amsterdam: Department of Psychological Methods, Amsterdam, The Netherlands
²Leiden University: Department of Clinical Psychology, Leiden, The Netherlands

[Download preprint](#)

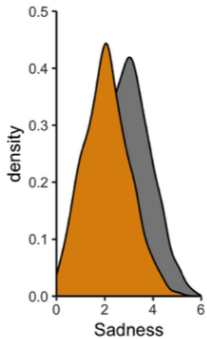
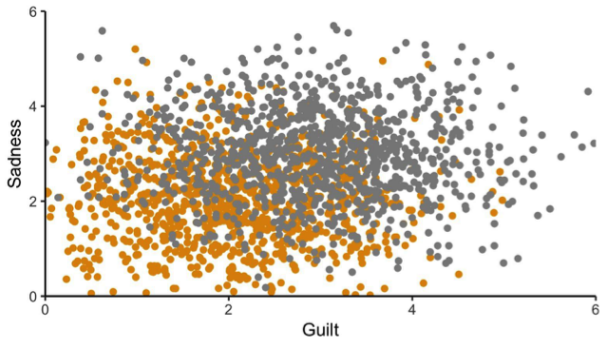
Downloads: 423

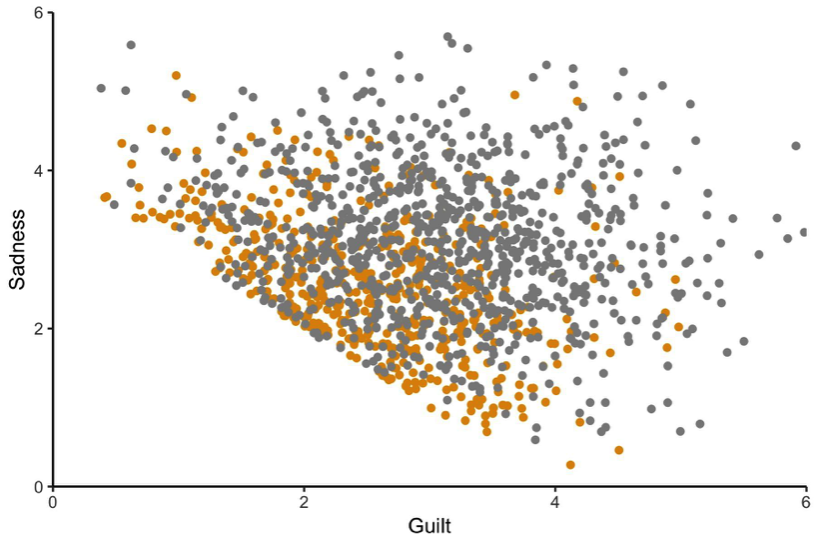
**Abstract**

In clinical research, populations are often selected on the sum-score of diagnostic criteria, i.e. symptoms. Estimating statistical models where a subset of the data is selected based on a function of the analyzed variables introduces Berkson's bias, which presents a potential threat to the validity of findings in the clinical literature. The aim ...

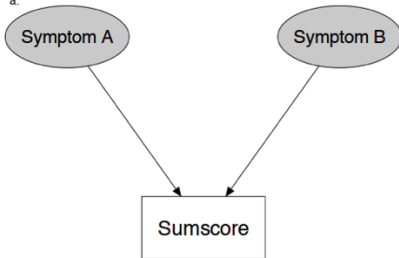
[See more](#)**Preprint DOI**[10.21203/psyarxiv.5t8zw](https://doi.org/10.21203/psyarxiv.5t8zw)

<https://psyarxiv.com/5t8zw/>

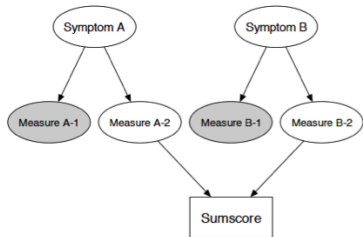




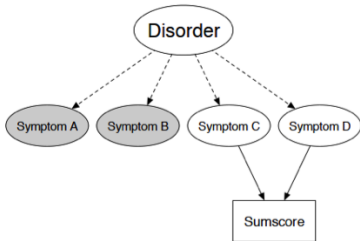
a.



b.



c.



d.

