SEM 2: Structural Equation Modeling

Week 3 - Causality & DAGs

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- If we see that you have a high heart rate, we can predict that you are exercising

$\mathrm{Exercising} \rightarrow$ Higher heart rate

Implies:

- Observing that you are exercising makes it more likely that you have a higher heart rate
  - $\mathcal{E}(\text{heart rate} \mid \mathrm{See}(\text{exercising})) > \mathcal{E}(\text{heart rate})$

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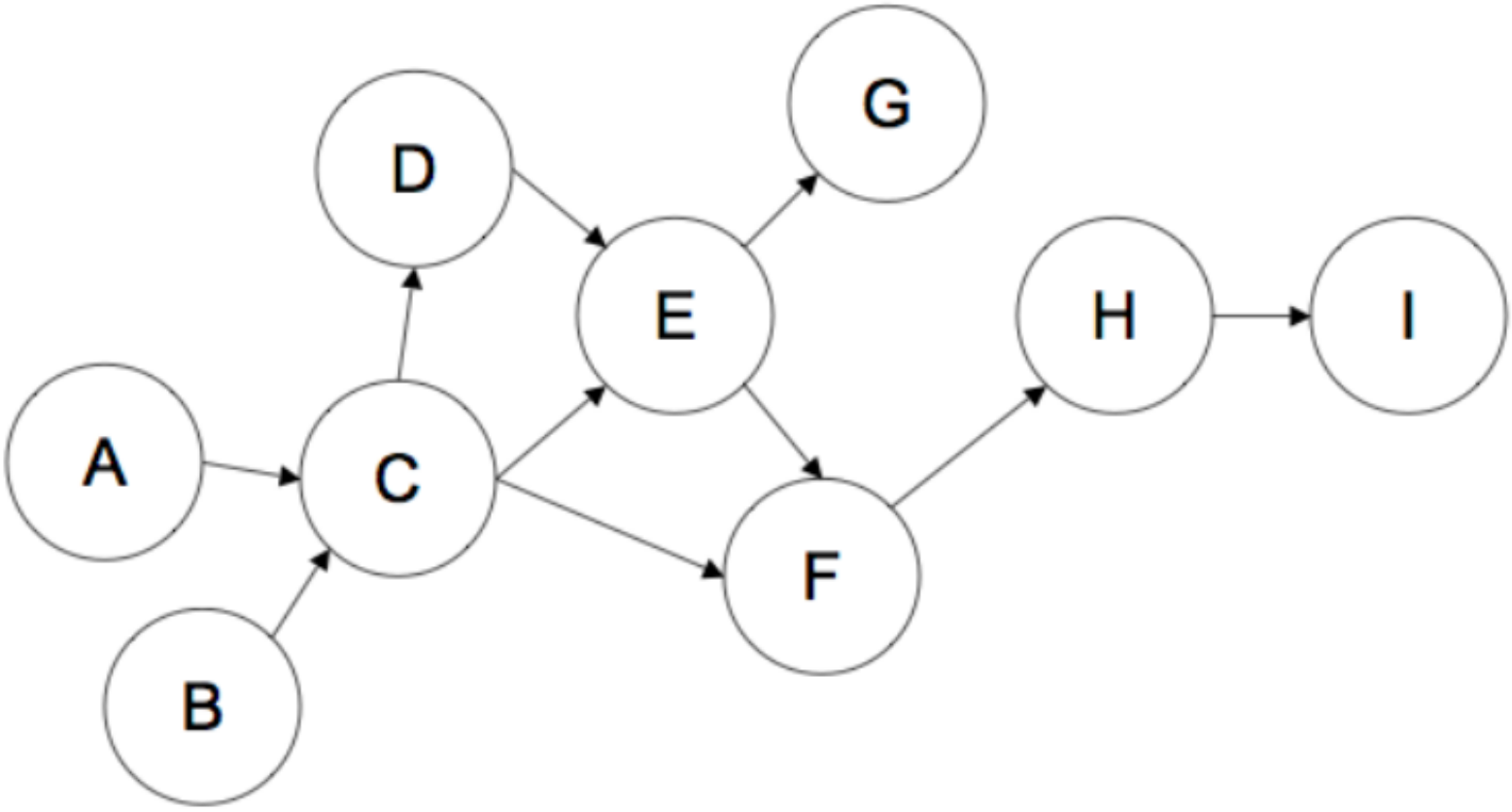
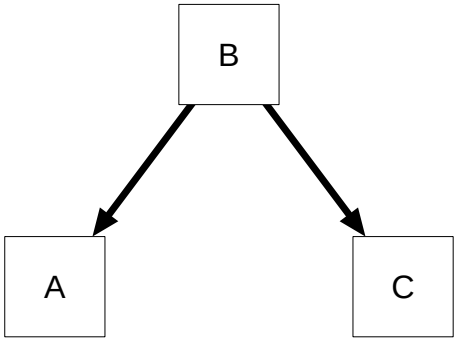
Solution: More variables and more advanced causal models imply more testable hypotheses (conditional independence relations)!

Directed Acyclic Graphs



Building blocks of a DAG

Common Cause

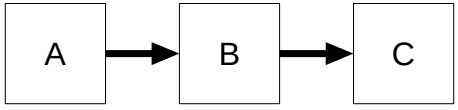


Example: Disease (B) causes two symptoms (A and C).

$A \not\perp\!\!\!\perp C$

$A \perp\!\!\!\perp C \mid B$

Chain



Example: Insomnia (A) causes fatigue (B), which in turn causes concentration problems (C)

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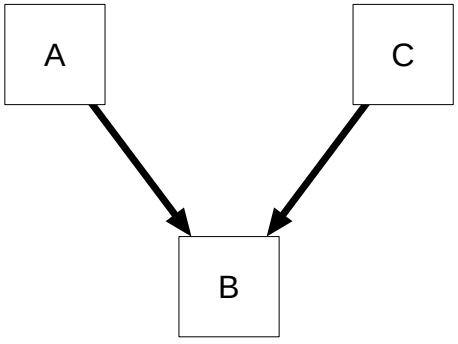
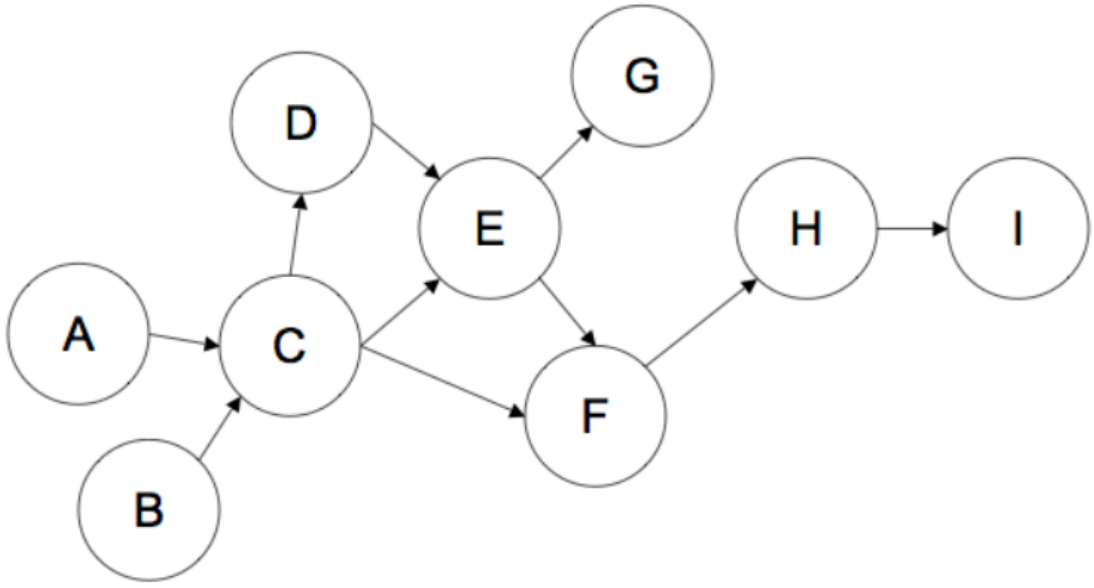
Collider



Example: Difficulty of class (A) and motivation of student (C) cause grade on a test (B)

$A \perp\!\!\!\perp C$

$A \not\perp\!\!\!\perp C \mid B$



- $A \perp\!\!\!\perp B$
- $A \perp\!\!\!\perp D \mid C$
- $B \perp\!\!\!\perp G \mid C, E$
- ...

Testing this causal model involves testing if all these conditional independence relations hold