SEM 2: Structural Equation Modeling
Week 3 - Causality & DAGs

Sacha Epskamp
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- If we see that you are exercising, we can predict your heart rate to be higher than average
- If we see that you have a high heart rate, we can predict that you are exercising
Exercising $\rightarrow$ Higher heart rate

Implies:

- Observing that you are exercising makes it more likely that you have a higher heart rate
  - $\mathcal{E}(\text{heart rate} \mid \text{See(exercising)}) > \mathcal{E}(\text{heart rate})$
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- Observing that you have a high heartrate makes it more likely you are exercising a lot
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- But making your heart rate high does not make you exercise!
  - $\mathcal{E}(\text{exercising} \mid \text{Do(high heart rate)}) = \mathcal{E}(\text{exercising})$

Unfortunately, in observational data (especially without temporal ordering), we can only investigate what happens if we see one variable (conditioning)...

Solution: More variables and more advanced causal models imply more testable hypotheses (conditional independence relations)!
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Directed Acyclic Graphs
Building blocks of a DAG

Common Cause

Example: Disease (B) causes two symptoms (A and C).

A ∟ C
A ⊥ C | B

Chain

Example: Insomnia (A) causes fatigue (B), which in turn causes concentration problems (C)

A ∟ C
A ⊥ C | B

Collider

Example: Difficulty of class (A) and motivation of student (C) cause grade on a test (B)

A ⊥ C
A ∟ C | B
Testing this causal model involves testing if all these conditional independence relations hold:

- $A \perp B$
- $A \perp D \mid C$
- $B \perp G \mid C, E$
- ...

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