Assignment 2

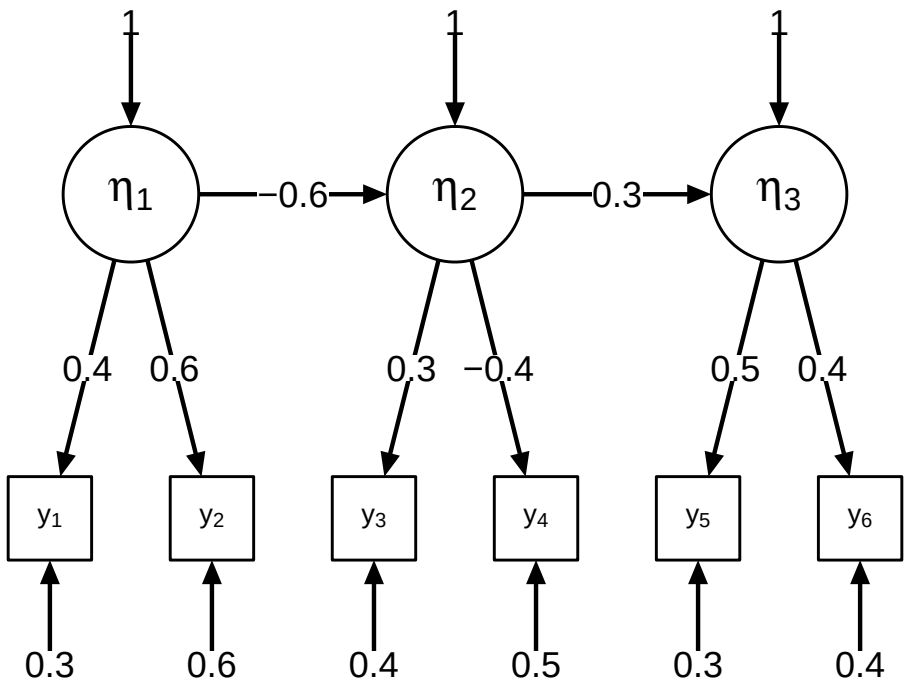
SEM 2: Structural Equation Modeling

Please hand in a `.pdf` file containing your report and a `.R` containing your codes or screenshots of every Jasp analysis. The deadline of this assignment is Thursday May 14 13:00.

Assignment

Part 1. Modeling SEMs

Consider the following model:



Question 1 (1 point) Give the matrices $\boldsymbol{\Lambda}$, $\boldsymbol{B}$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Theta}$ (containing values, not symbols). ■

Question 2 (1 point) Compute $\boldsymbol{\Sigma}$ using the all-y formula from the slides. Note that $\boldsymbol{I}$ is an identity matrix, a matrix with ones on the diagonal and zeroes elsewhere, of the same dimensions as $\boldsymbol{B}$. ■

Question 3 (1 point) Use Wright's path tracing rules to determine $\text{var}(y_6)$, and compare your answer to Question 2. ■

Question 4 (1 point) Use Wright's path tracing rules to determine $\text{cov}(y_1, y_6)$, and compare your answer to Question 3. ■

It is known that $(\boldsymbol{I}-\boldsymbol{B})^{-1}$ can be written as:

$$\begin{aligned}(\boldsymbol{I}-\boldsymbol{B})^{-1} &= \boldsymbol{I}+\boldsymbol{B}+\boldsymbol{B}^2+\boldsymbol{B}^3+\boldsymbol{B}^4+\ldots \\ &= \boldsymbol{I}+\boldsymbol{B}+\boldsymbol{BB}+\boldsymbol{BBB}+\boldsymbol{BBBB}+\ldots\end{aligned} \tag{1}$$

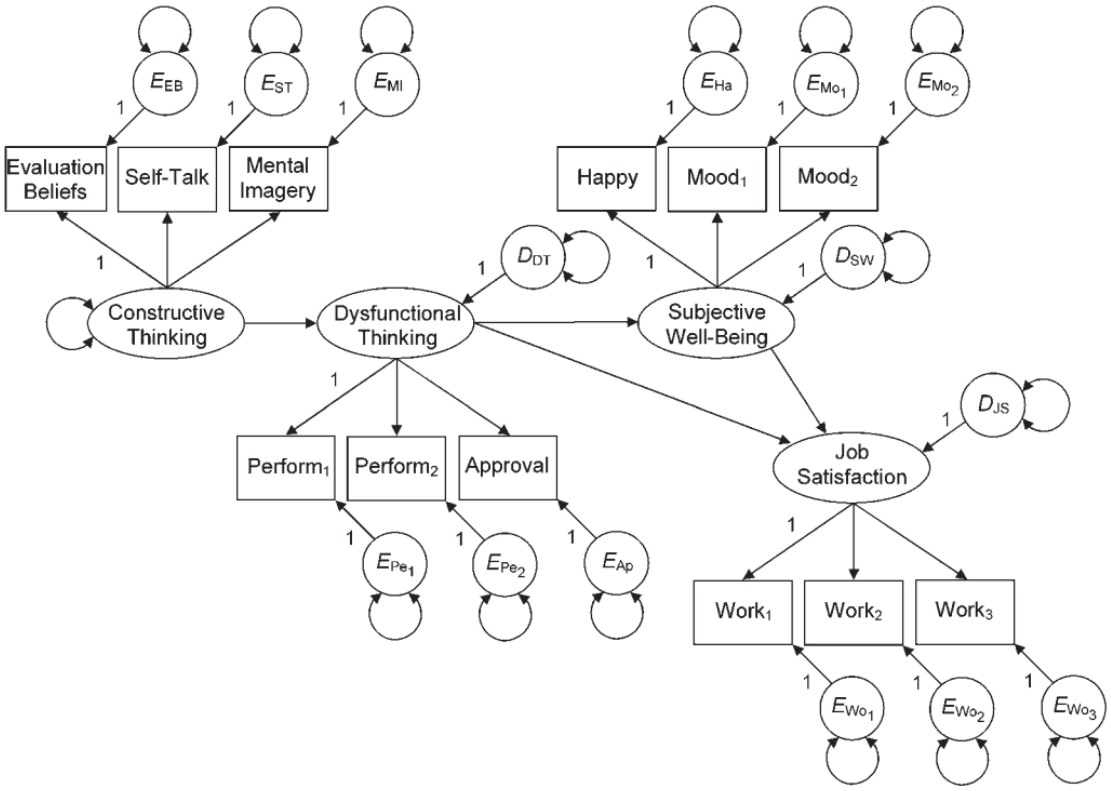
if $\boldsymbol{B}$ is *nilpotent*, meaning that for some finite k, $\boldsymbol{B}^k = \boldsymbol{O}$ (a matrix containing only zeroes) and as a result for every $l \in \mathbb{Z}^+$ (l is a positive integer, such as 1, 2 or 3), $\boldsymbol{B}^{k+l} = \boldsymbol{O}$.

Question 5 (1 point) Show that $\boldsymbol{B}$ is nilpotent for $k \geq 3$ and compute $(\boldsymbol{I}-\boldsymbol{B})^{-1}$ using Equation (1). Also compute $(\boldsymbol{I}-\boldsymbol{B})^{-1}$ using R and compare your results. ■

Question 6 (1 point) This model is identified due to (in part) fixing diagonal values of $\boldsymbol{\Psi}$ to 1. In CFA, this means that the variance of each latent (η) is fixed to 1. Is this also the case for more general SEMs? such as the one shown above? Explain your answer. ■

Part 2. Fitting SEMs

Kline (2015) reports a path diagram of a SEM analysis that was originally performed by Houghton and Jinkerson (2007):



Question 7 (1 point) Verify that the degrees of freedom in this model are 50 (note: the original authors report DF = 51, which seems to result from some mistake made in the analysis). ■

The original article contains a correlation matrix, standard deviations, and the sample size ($N = 263$). With this information, we can also construct the variance–covariance matrix, which I prepared for you in the file `houghton.csv` on Canvas, which can be loaded in R as follows:

```
covMat <- as.matrix(read.csv("houghton.csv"))
rownames(covMat) <- colnames(covMat)
```

Question 8 (2 points) Fit the SEM model to the data (1 point), and judge the fit of your model (1 point). Note, the original article will report something different, likely due to the same mistake that led to the wrong degrees of freedom. ■

Question 9 (1 point) We may be interested to test if subjective well-being fully mediates the effect of dysfunctional thinking on job satisfaction. To this end, we can fix the direct path from dysfunctional thinking to job satisfaction to zero, and compare the fit of the restricted model to the general model you tested above. Can you conclude that subjective well-being fully mediates the effect between dysfunctional thinking and job satisfaction? ■

References

Houghton, J. D., & Jinkerson, D. L. (2007). Constructive thought strategies and job satisfaction: A preliminary examination. *Journal of Business and Psychology*, *22*(1), 45–53.

Kline, R. B. (2015). *Principles and practice of structural equation modeling*. Guilford publications.