SEM and GGMs

SEM 2 2020

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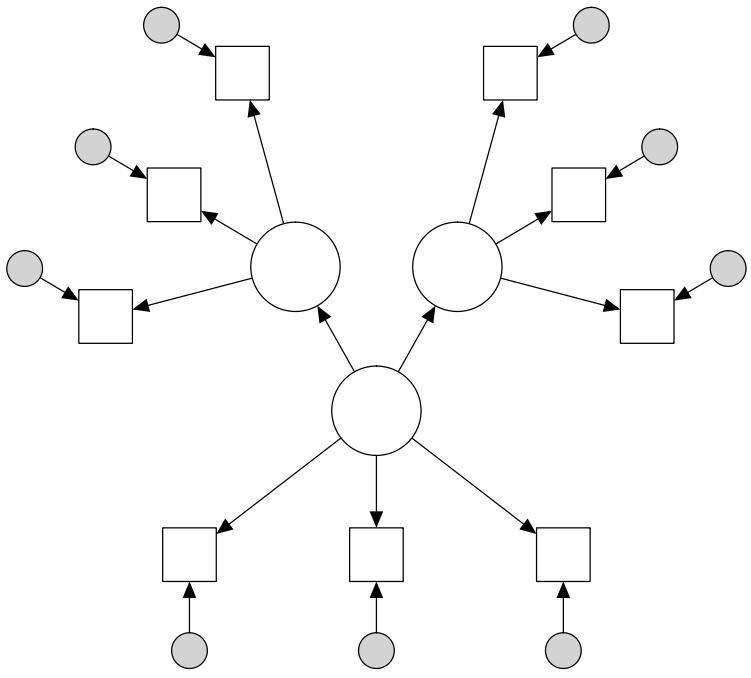
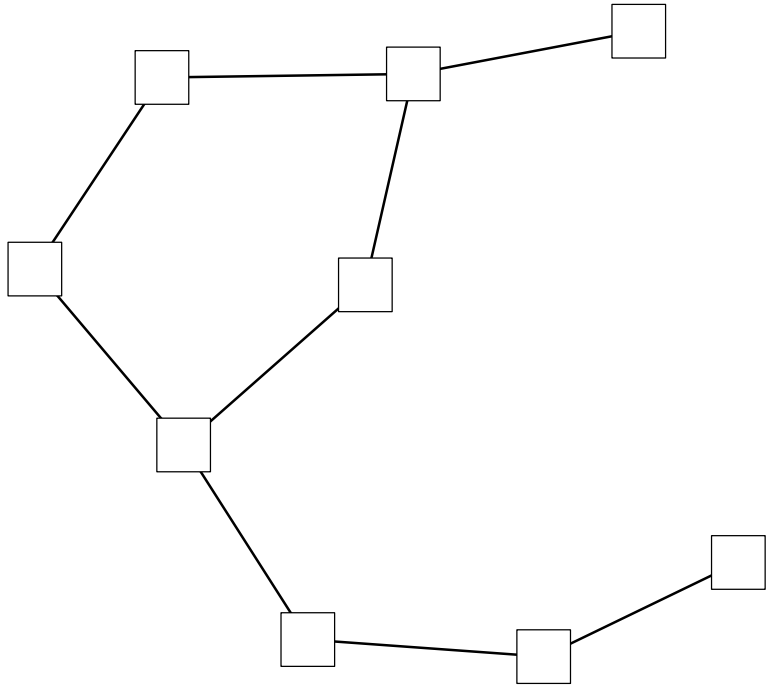
SEM and GGM

Structural Equation Modeling



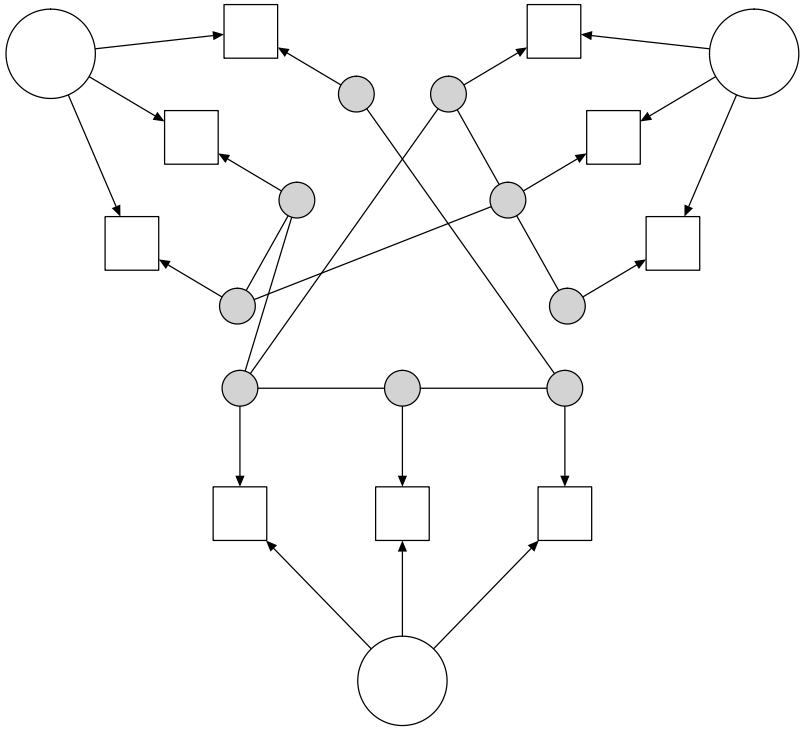
Gaussian Graphical Model



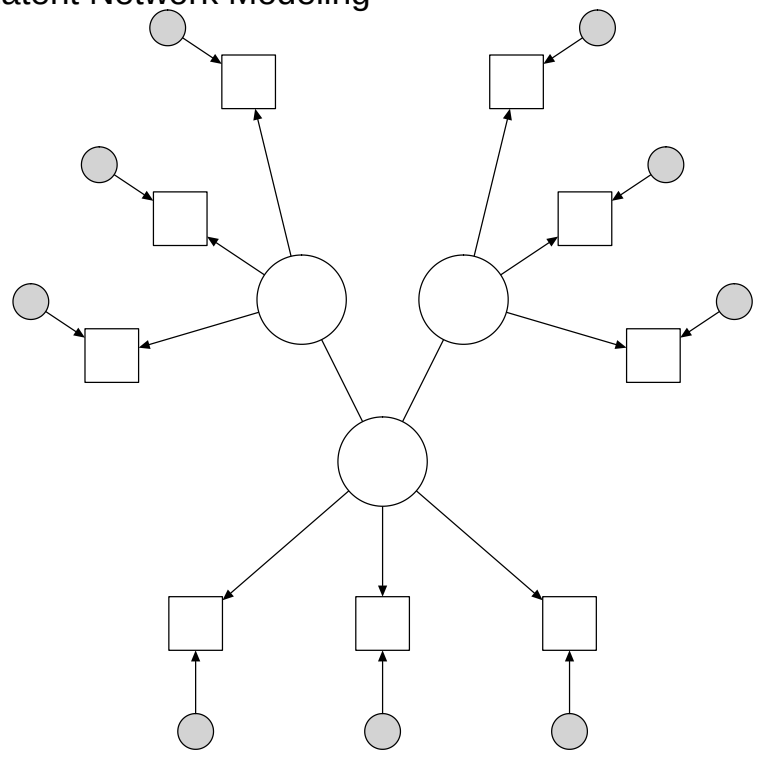
- Both models imply a variance-covariance matrix $\boldsymbol{\Sigma}$, aimed to closely resemble the sample variance-covariance matrix $\boldsymbol{S}$ with positive degrees of freedom.

Augment Structural Equation Models (SEM) by modeling either the residuals or latent covariances as a Gaussian Graphical model (GGM):

Residual Network Modeling



Latent Network Modeling



Implementing networks in SEM

The variance-covariance matrices in a SEM model can be modeled as a network. Using a CFA model as starting point:

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Psi}\boldsymbol{\Lambda}^{\top} + \boldsymbol{\Theta}$$

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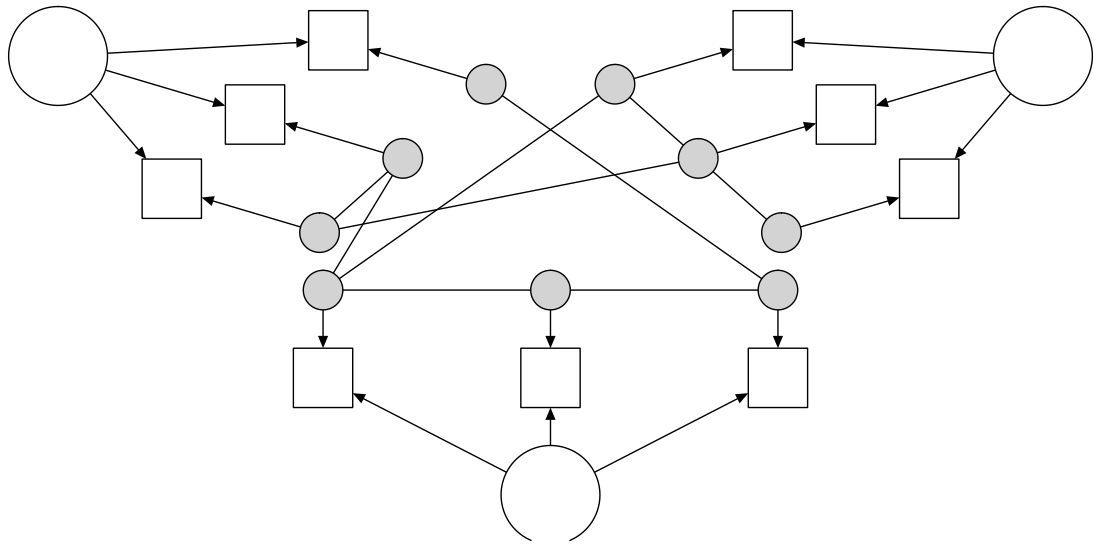
Or the latent level:

$$\boldsymbol{\Psi} = \boldsymbol{\Delta}_{\boldsymbol{\Psi}} \left(\boldsymbol{I} - \boldsymbol{\Omega}_{\boldsymbol{\Psi}}\right)^{-1} \boldsymbol{\Delta}_{\boldsymbol{\Psi}}$$

Residual Network Modeling (RNM)

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Psi}\boldsymbol{\Lambda}^{\top} + \boldsymbol{\Delta}_{\boldsymbol{\Theta}}\left(\boldsymbol{I} - \boldsymbol{\Omega}_{\boldsymbol{\Theta}}\right)^{-1}\boldsymbol{\Delta}_{\boldsymbol{\Theta}}$$

- Network is formed at the residuals of SEM
- Model a network while not assuming no unobserved common causes
- Model a latent variable structure without the assumption of local independence



Latent Network Modeling (LNM)

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Delta}_{\boldsymbol{\psi}} \left(\boldsymbol{I} - \boldsymbol{\Omega}_{\boldsymbol{\psi}}\right)^{-1} \boldsymbol{\Delta}_{\boldsymbol{\psi}}\boldsymbol{\Lambda}^{\top} + \boldsymbol{\Theta}$$

- Models conditional independence relations between latent variables as a network
- Model networks between latent variables
- Exploratory search for conditional independence relationships between latents