SEM 2: Structural Equation Modeling

Week 2 - Path analysis

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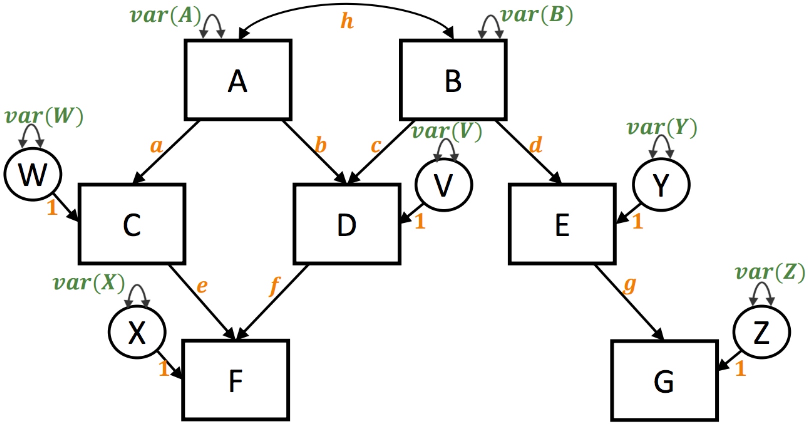
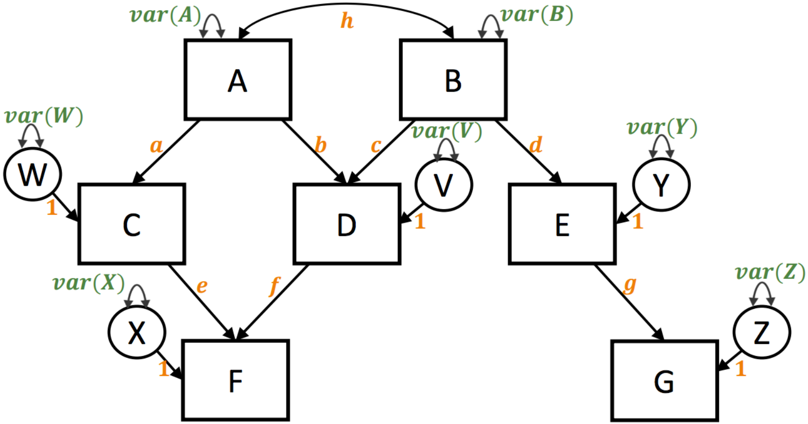
May 1, 2020



How many degrees of freedom?



How many degrees of freedom?

- $7 \times 8/2 = 28$ observed variances and covariances
- 7 regressions $+7$ variances $+1$ covariance $= 15$ parameters
- 13 degrees of freedom

Wright's path tracing Rules

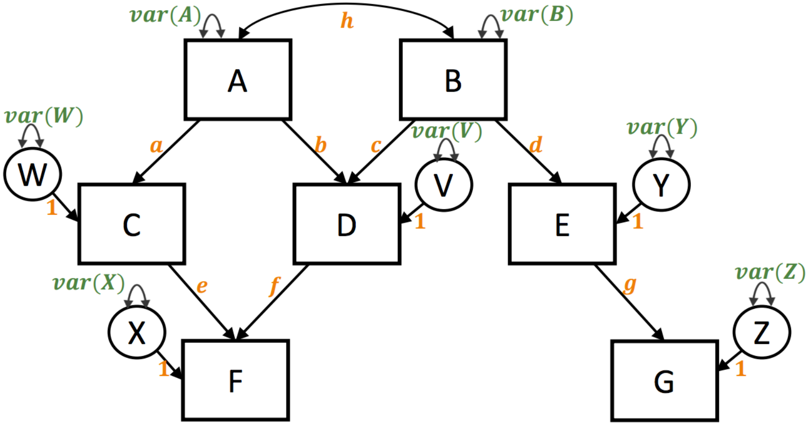
The covariance between any two variables can be expressed as the sum of the compound paths connecting them.

To obtain a compound path:

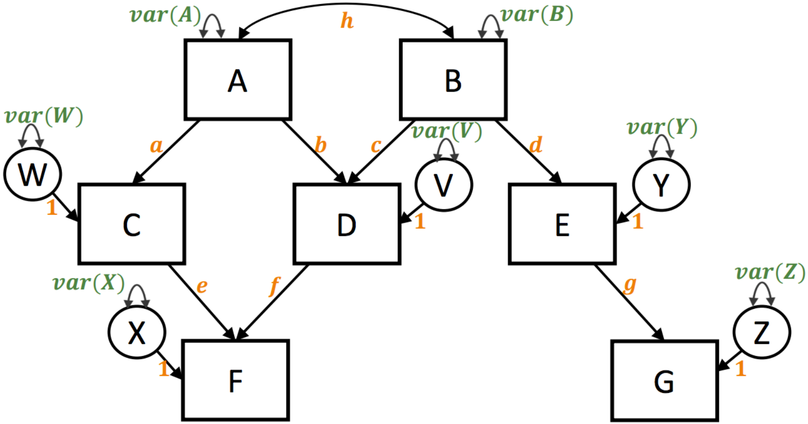
- Trace backwards, change direction at a two-headed arrow, then trace forwards
- Do not go forward and then backward
  - You can never pass out of one arrow head and into another arrowhead: heads-tails, or tails-heads, not heads-heads
- Must contain one, and only one, variance or covariance (bidirectional edge)

Then to obtain the implied (co)variance:

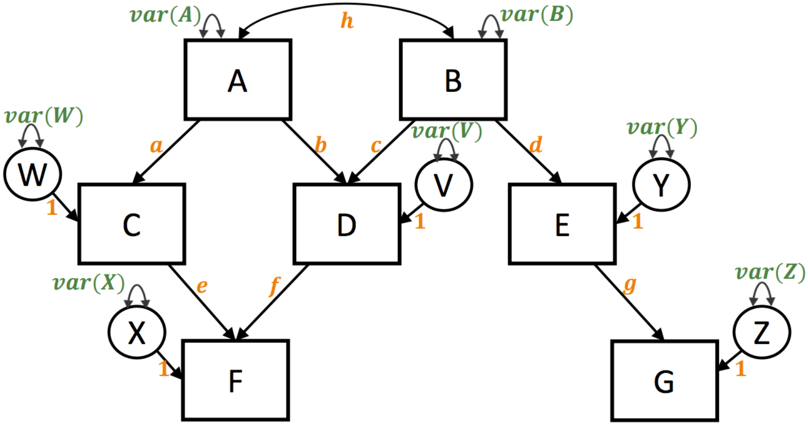
- Compute the product of coefficients in each route between the variables of interest
- Sum over all distinct routes, where pathways are considered distinct if they contain different coefficients, or encounter those coefficients in a different order



Compound paths between A and B:

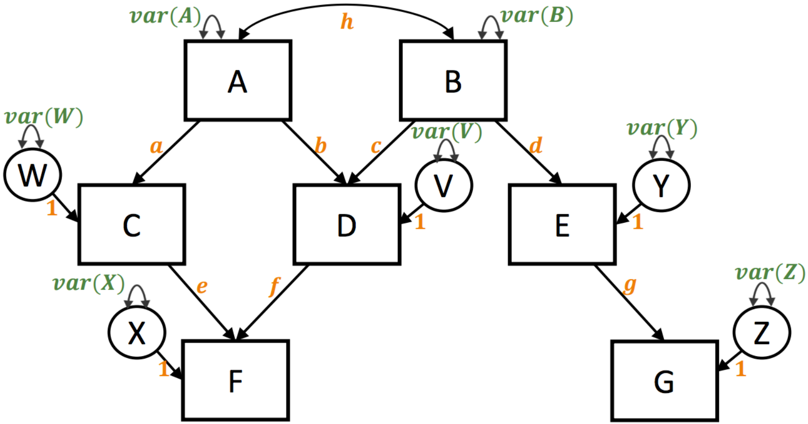


Compound paths between A and B: $A \leftrightarrow B$

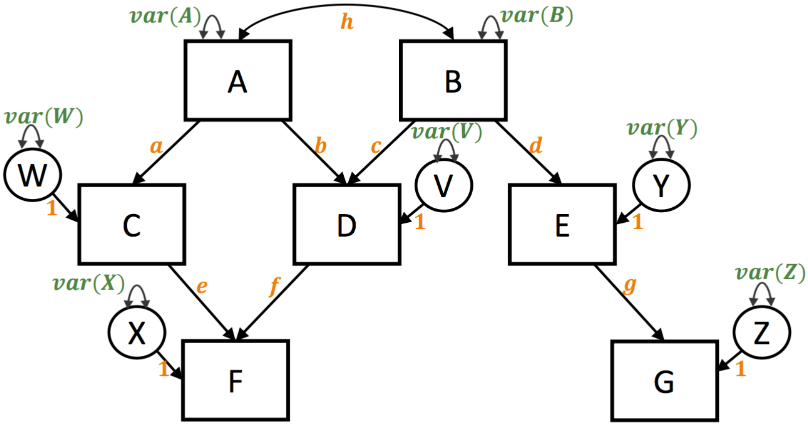


Compound paths between A and B: $A \leftrightarrow B$

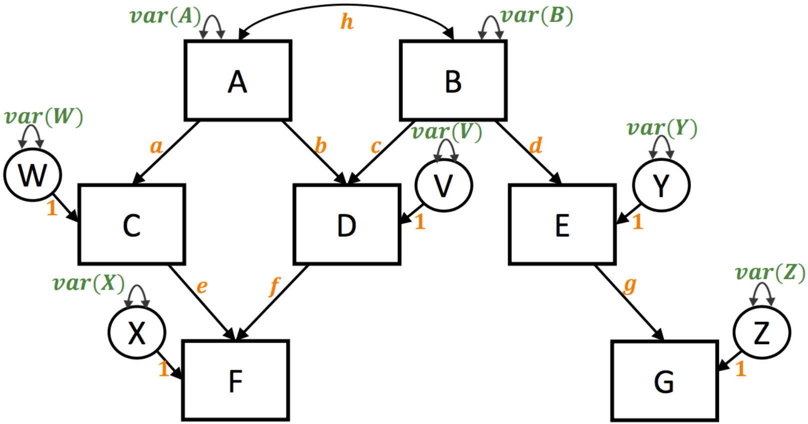
$$\mathrm{Cov}\,(A, B) = h$$



Compound paths between C and D:

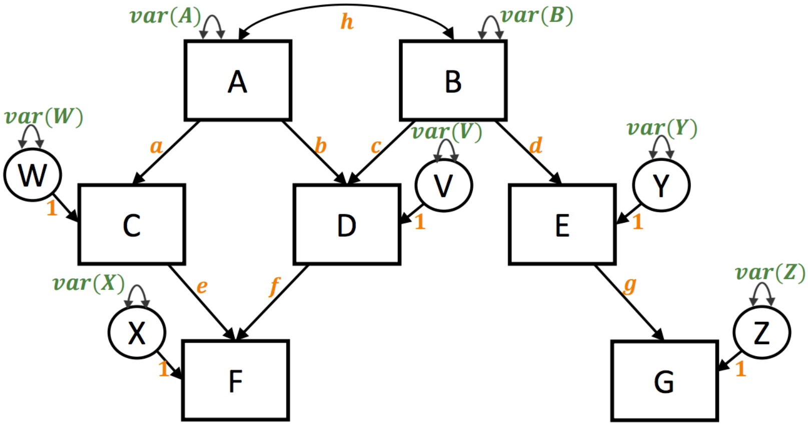


Compound paths between C and D: $C \leftarrow A \leftrightarrow A \rightarrow D$ and $C \leftarrow A \leftrightarrow B \rightarrow D$

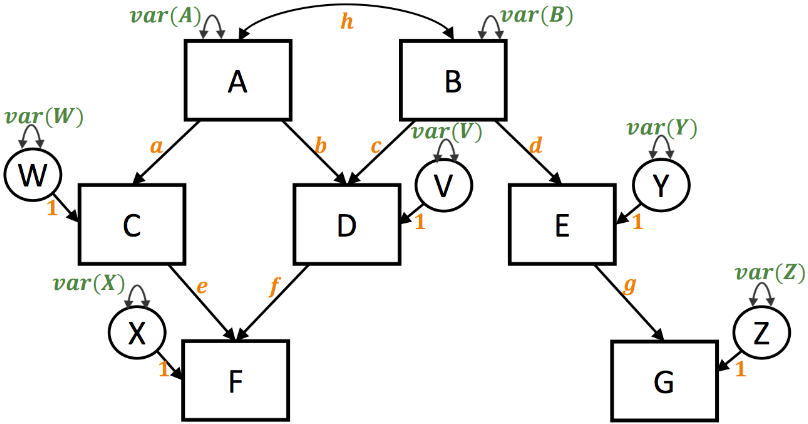


Compound paths between C and D: $C \leftarrow A \leftrightarrow A \rightarrow D$ and $C \leftarrow A \leftrightarrow B \rightarrow D$

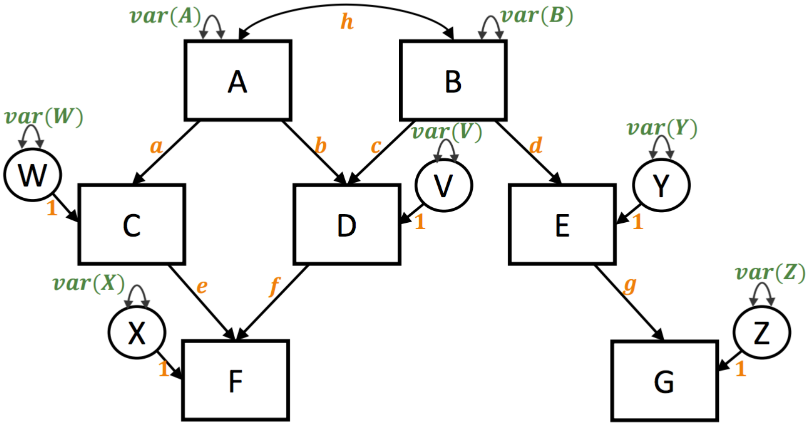
$$\mathrm{Cov}\,(C, D) = a(\mathrm{var}(A))b + ahc$$



Compound paths between C and itself:

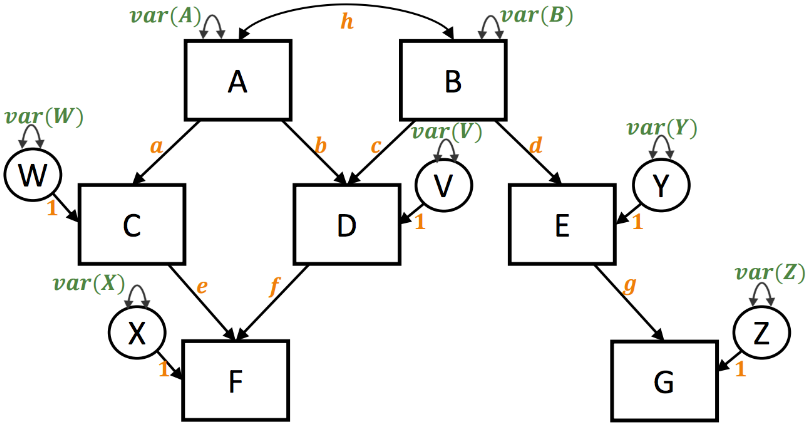


Compound paths between C and itself: $C \leftarrow W \leftrightarrow W \rightarrow C$ and $C \leftarrow A \leftrightarrow A \rightarrow C$

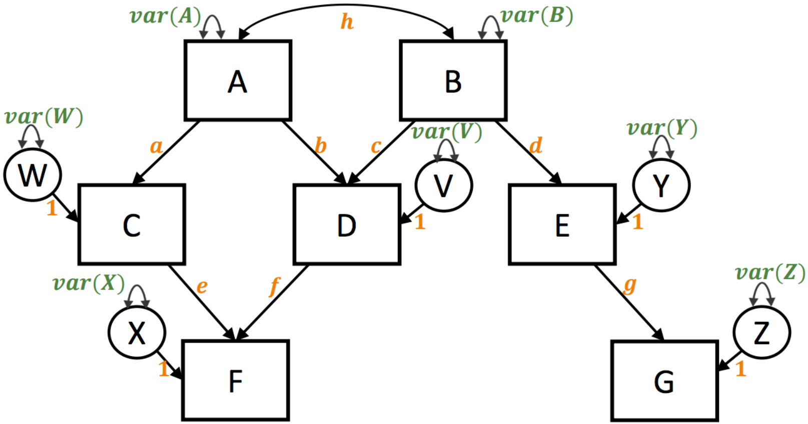


Compound paths between C and itself: $C \leftarrow W \leftrightarrow W \rightarrow C$ and $C \leftarrow A \leftrightarrow A \rightarrow C$

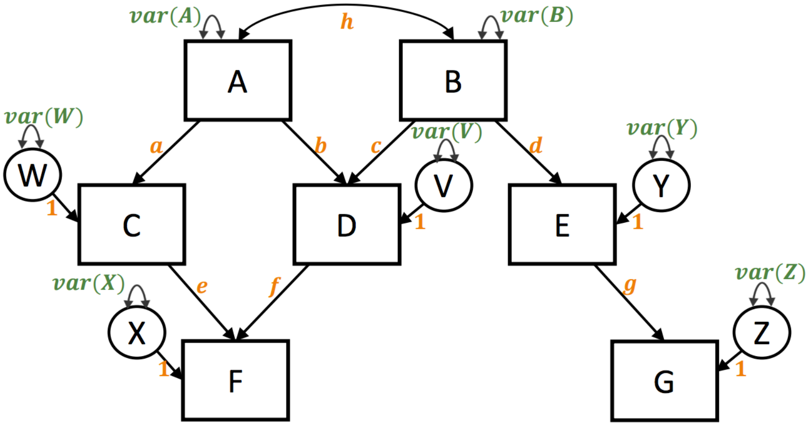
$$\mathrm{Var}\,(C) = a^2(\mathrm{var}(A)) + \mathrm{Var}(W)$$



Compound paths between F and G:



Compound paths between F and G: $F \leftarrow D \leftarrow B \leftrightarrow B \rightarrow E \rightarrow G$, $F \leftarrow C \leftarrow A \leftrightarrow B \rightarrow E \rightarrow G$, and $F \leftarrow D \leftarrow A \leftrightarrow B \rightarrow E \rightarrow G$.



Compound paths between F and G: $F \leftarrow D \leftarrow B \leftrightarrow B \rightarrow E \rightarrow G$, $F \leftarrow C \leftarrow A \leftrightarrow B \rightarrow E \rightarrow G$, and $F \leftarrow D \leftarrow A \leftrightarrow B \rightarrow E \rightarrow G$.

$$\mathrm{Cov}\left(F, G\right) = fc(\mathrm{Var}(B))dg + fbhdg + eahgd$$