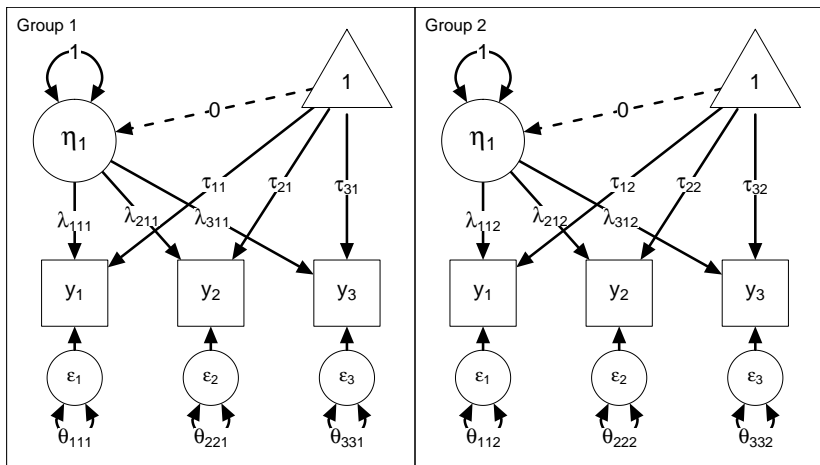


# SEM 1: Confirmatory Factor Analysis

Week 3 - Measurement invariance

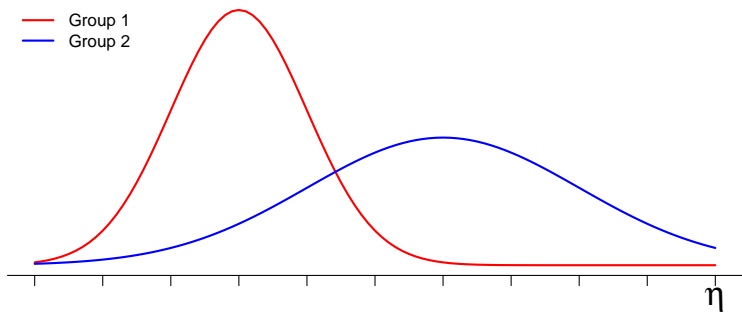
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2020



3 means, 3 variances and 3 covariances per group = 18 observations. 3 residual variances; 3 factor loadings and 3 intercepts per group = 18 parameters; DF = 0.

The goal of multi-group CFA is to measure differences in means and (co)variances (networks) between groups:



That is, we wish to perform tests for *homogeneity*:

- ▶  $\alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha$
- ▶  $\psi_1 = \psi_2 = \psi_3 = \dots = \psi$

Without measurement error, that would be similar MANOVA, Box's test, Levene's test, etcetera.

But...

- ▶ The latent means are not identified
- ▶ The latent variances are arbitrary due to scaling
- ▶ We don't know if the test measures the same constructs in both groups.

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It turns out these problems can both be solved by imposing sequential equality constraints and testing for increasing levels of **measurement invariance**:

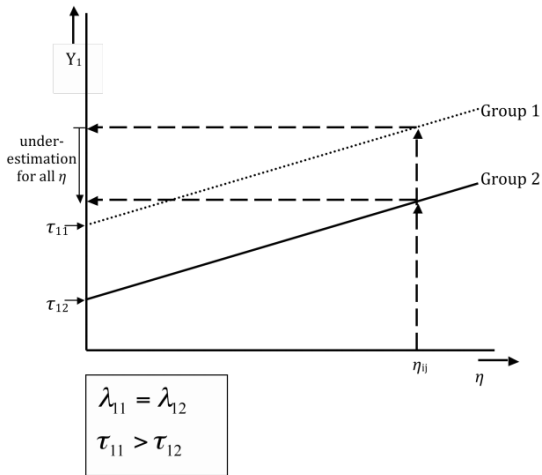
Name	Additional constraints	Allows to test
Configural invariance	Same zeroes in $\Lambda_1, \Lambda_2, \dots$	
Weak invariance	$\Lambda_1 = \Lambda_2 = \dots = \Lambda$	$\Psi_1 = \Psi_2 = \dots = \Psi$
Strong invariance	$\tau_1 = \tau_2 = \dots = \tau$	$\alpha_1 = \alpha_2 = \dots = \alpha$
Strict invariance	$\Theta_1 = \Theta_2 = \dots = \Theta$	Full homogeneity

Note:  $\alpha_2, \alpha_3, \dots$  are identified if strong invariance holds.

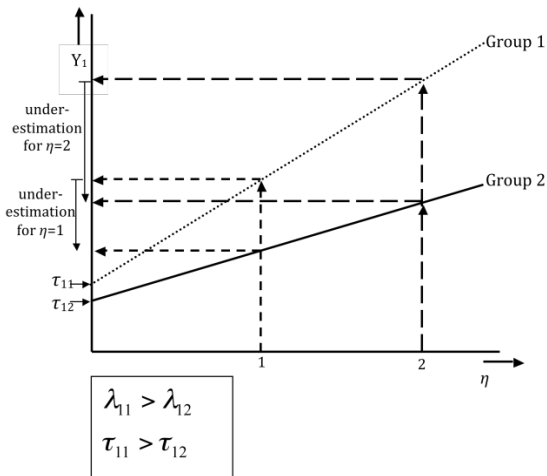
## Measurement Invariance

- ▶ To compare two groups on their mean or variance, it is imperative that the test **measures the same construct** in both groups, and is **not biased**.
  - ▶ Group membership may influence the latent trait, but should not influence individual items.
- ▶ If measurement invariance does not hold, then test scores (e.g., sum-scores) can **not** be used to compare groups in a meaningful way
- ▶ If **partial invariance** holds (most parameters are equal across groups), multi-group CFA can be used to test for homogeneity in means and variances
  - ▶ But interpretation gets harder the more measurement invariance is violated!

Canonical reference: Mellenbergh, G. J. (1989). Item bias and item response theory. *International journal of educational research*, 13(2), 127-143.

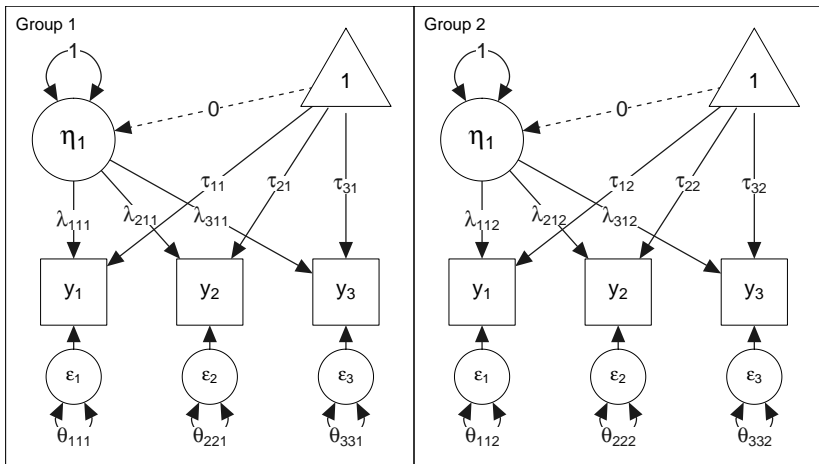


Wicherts, J. M., & Dolan, C. V. (2010). Measurement invariance in confirmatory factor analysis: An illustration using IQ test performance of minorities. *Educational Measurement: Issues and Practice*, 29(3), 39-47.



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Configural invariance: Does the same model fit in both groups?

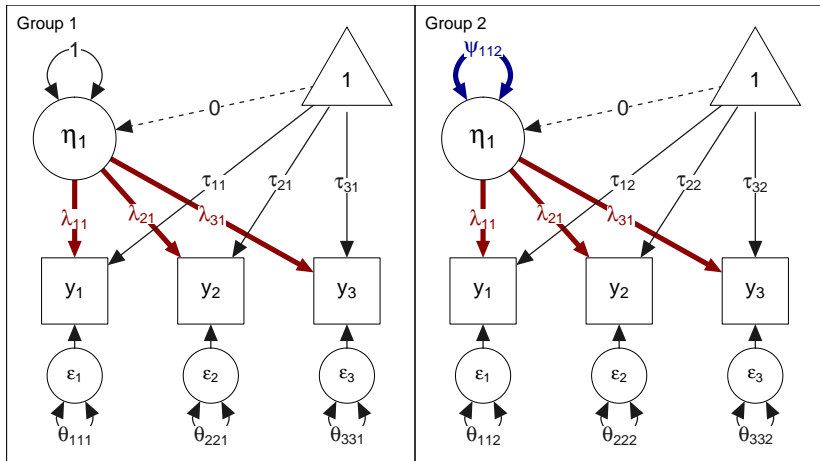
```
# Load the package:
library("lavaan")

# Load data:
data("HolzingerSwineford1939")
Data <- HolzingerSwineford1939

# Model:
Model <- '
  visual  =~ x1 + x2 + x3 + x9
  textual =~ x4 + x5 + x6
  speed   =~ x7 + x8 + x9
  x3 ~~ x5
  '

# Fit configural:
conf <- cfa(Model, Data, group = "school")
fitMeasures(conf,
             c("rmsea", "cfi", "tli", "rni", "rfi", "ifi", "srmr", "gfi"))

## rmsea  cfi  tli  rni  rfi  ifi  srmr  gfi
## 0.069 0.964 0.942 0.964 0.871 0.965 0.049 0.997
```



Weak Invariance: Are factor loadings the same? — Differences in variance (and covariances) of latent variable now interpretable!

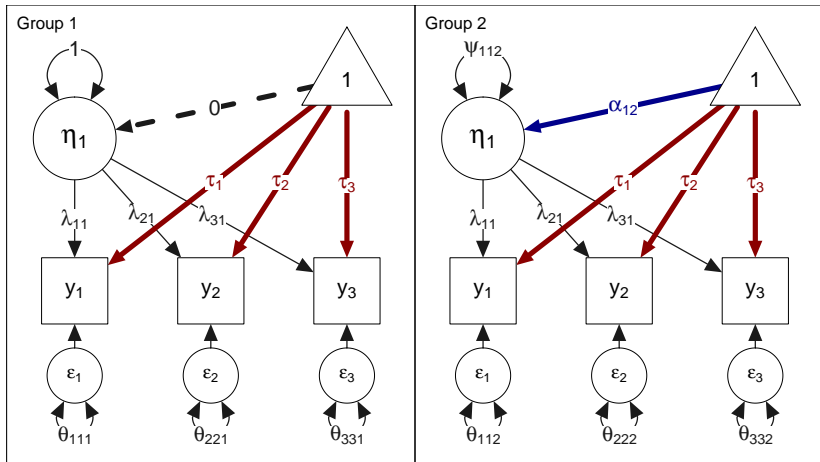
```

weak <- cfa(Model, Data, group = "school",
            group.equal = "loadings")
anova(conf, weak)

## Chi-Squared Difference Test
##
##      Df      AIC      BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## conf  44 7452.2 7689.4 75.646
## weak  51 7444.6 7655.9 82.094      6.4487      7      0.4884

```

(7 DF difference:  $-7$  free factor loadings)



Strong Invariance: Are the intercepts the same? — Latent mean difference now identified!

```

strong <- cfa(Model, Data, group = "school",
              group.equal = c("loadings", "intercepts"))
anova(conf, weak, strong)

## Chi-Squared Difference Test
##
##           Df      AIC      BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## conf      44 7452.2 7689.4  75.646
## weak      51 7444.6 7655.9  82.094      6.449      7      0.4884
## strong    57 7473.2 7662.2 122.638     40.544      6 3.561e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

(6 DF difference:  $-9$  intercepts, but  $+3$  latent variable means)

Where is the misfit?

```
# Group 1:
lavInspect(strong, "mu")[[1]] -
lavInspect(strong, "sampstat")[[1]]$mean

##      x1      x2      x3      x9      x4      x5      x6      x7      x8
## 0.053 0.159 -0.227 0.051 -0.035 0.066 0.009 -0.182 0.077

# Group 2:
lavInspect(strong, "mu")[[2]] -
lavInspect(strong, "sampstat")[[2]]$mean

##      x1      x2      x3      x9      x4      x5      x6      x7      x8
## -0.057 -0.130 0.166 -0.041 0.028 -0.032 -0.015 0.132 -0.064
```

Some other options are possible (e.g., 'lavTestScore' in lavaan).

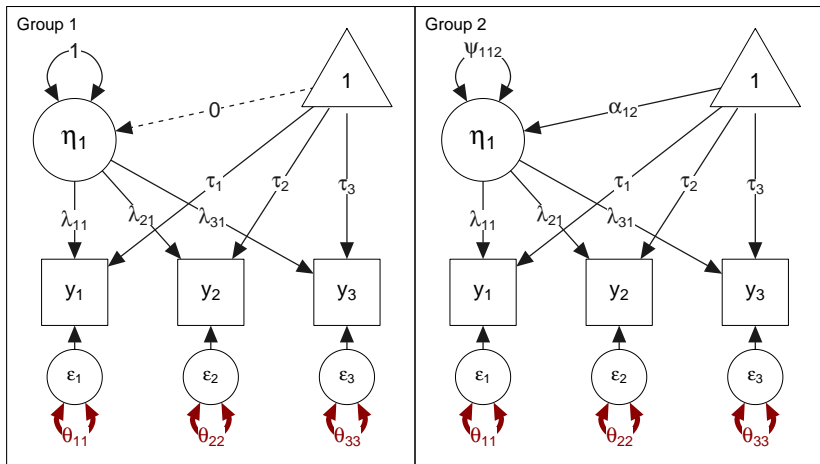
## Free two intercepts:

```
strong2 <- cfa(Model, Data, group = "school",
  group.equal = c("loadings", "intercepts"),
  group.partial = c("x3 ~ 1", "x7 ~ 1"))

anova(conf, weak, strong, strong2)

## Chi-Squared Difference Test
##
##           Df      AIC      BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## conf      44 7452.2 7689.4  75.646
## weak      51 7444.6 7655.9  82.094      6.449      7      0.4884
## strong2   55 7442.4 7638.9  87.846      5.752      4      0.2185
## strong    57 7473.2 7662.2 122.638     34.792      2 2.786e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```





Strict Invariance: Are the residual variances the same?

```

strict <- cfa(Model, Data, group = "school",
  group.equal = c("loadings","intercepts","residuals",
    "residual.covariances"),
  group.partial = c("x3 ~ 1","x7 ~ 1"))
anova(strong2, strict)

## Chi-Squared Difference Test
##
##           Df      AIC      BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## strong2  55 7442.4 7638.9  87.846
## strict   65 7441.9 7601.3 107.398    19.552      10    0.03379 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

10 less parameters (residual variances). Strict invariance holds adequately (although debatable).