Assignment 1

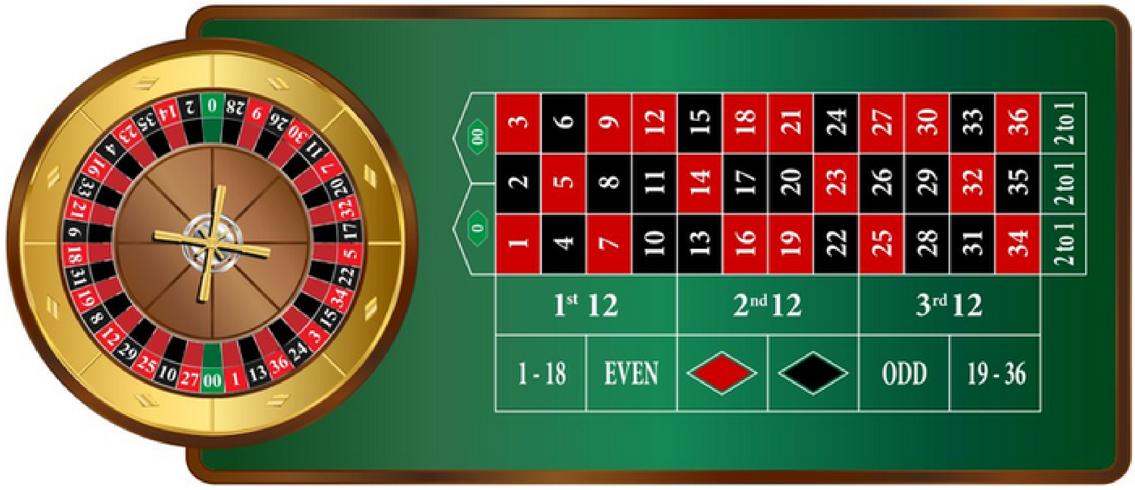
SEM 2: Structural Equation Modeling

Please hand in a `.pdf` file containing your report and a `.R` containing your codes or screenshots of every Jasp analysis. The deadline of this assignment is Tuesday May 15 15:00.

Assignment

Part 1: Expectated values

The *American Roulette* casino game is slightly different from the regular (French) roulette game described in the lecture, in that an extra "00" field is added, leading to a total of 38 spaces the ball can fall on:



Betting € 1 on the right number still leads to a profit of € 35, and betting € 1 on the wrong number still leads to losing the Euro.

Question 1 (1 point) What is the expected value of betting € 1 on the number '15'? Is this better or worse than betting € 1 on the number '15' in French roulette? ■

Instead of betting only on one number, we could 'spread our chances' by betting on multiple numbers. For example, we can bet € 1 on "15", "20" and "25" each (betting € 3 total).

Question 2 (1 point) What is the expected value of this betting € 1 on "15", "20" and "25" each? Is it better to bet € 1 on three numbers in a single game rather than betting € 1 on one number in three consecutive game?
Tip: Note that by betting on three numbers in a single game, we are already sure to lose at least two out of three bets, as only one number can win! ■

Part 2: Covariance algebra

Given the following structural equations:

$$
\begin{aligned}
y_{i1} &= \beta_{21} y_{i2} + \beta_{31} y_{i3} + \varepsilon_{i1} \\
y_{i2} &= \beta_{42} x_i + \varepsilon_{i2} \\
y_{i3} &= \beta_{43} x_i + \varepsilon_{i3}
\end{aligned}
$$

Question 3 (1 point)
Draw the corresponding path diagram ■

Question 4 (1 point) Derive $\mathrm{Var}(y_1)$ and $\mathrm{Cov}(y_1, x)$ using covariance algebra. ■

A different modeling framework than the one used in class is the reticular action model (RAM). In RAM, the observed variables and latent variables are combined in one vector:

$$
\boldsymbol{v} = \begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{\eta} \end{bmatrix}
$$

Which is modeled using an *assymetric matrix* $\boldsymbol{A}$ (containing factor loadings and structural regressions) and a *symmetric matrix* $\boldsymbol{S}$ (containing exogenous and residual variances and covariances):

$$\boldsymbol{v}_i = \boldsymbol{A}\boldsymbol{v}_i + \boldsymbol{u}_i$$
$$\boldsymbol{u} \sim N(\boldsymbol{0}, \boldsymbol{S})$$

Question 5 (1 point) Derive an expression for $\text{Var}(\boldsymbol{v})$ in terms of $\boldsymbol{A}$ and $\boldsymbol{S}$.

Part 3: Fitting SEMs

In Good, Chavez & Sanchez (2010; on Blackboard), a model is specified in Figure 2 at the bottom of p. 458.

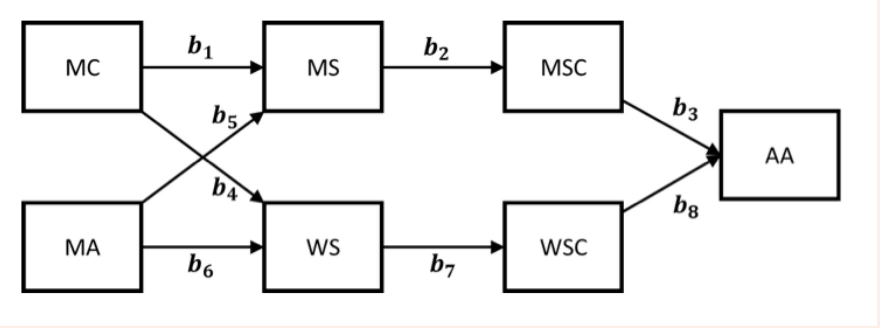
Question 6 (1 point) Fit this model using the variance–covariance matrix supplied in the paper. Assume that $N = 105$ (107 participants minus 2 eliminated for missing data). Also note that some paths are omitted from Figure 2: read footnote 4! Make sure that the model you run has the same degrees of freedom as theirs.

Question 7 (1 point) 7. Report the following values from your lavaan output and from the article:

	Lavaan output	Article
χ^2 (chi-square)		
p-value		

Question 8 (1 point) How many regressions, variances of exogenous variables and covariances between exogenous variables are there in the model? Compute the number of unique elements in the covariance matrix, and the degrees of freedom of the model.

Question 9 (1 point) The following model diagram contains the regression parameters in the model. Copy this (or draw the model yourself), add all other parameters in the model to the diagram, and label each parameter (e.g., var(MC)). Use curved double-sided arrows to display variances and covariances. Make sure you have the same number of parameters in total that you reported above.



Question 10 (2 points) Based on the obtained unstandardized path values, calculate (by hand) the model-implied (co)variances between the following variables, using covariance algebra. That is, solve the covariance algebra equation in symbolic form, to state the results in terms of model parameters. Then substitute each parameter with its estimated parameter value, and compute the answer. Compare your results with the values in the model-implied covariance matrix Lavaan gives (`lavInspect(fit, "sigma")`). Use the labels given in the path diagram above for paths and variable names.

1. Covariance between Minority Appearance and Minority Self-Categorization

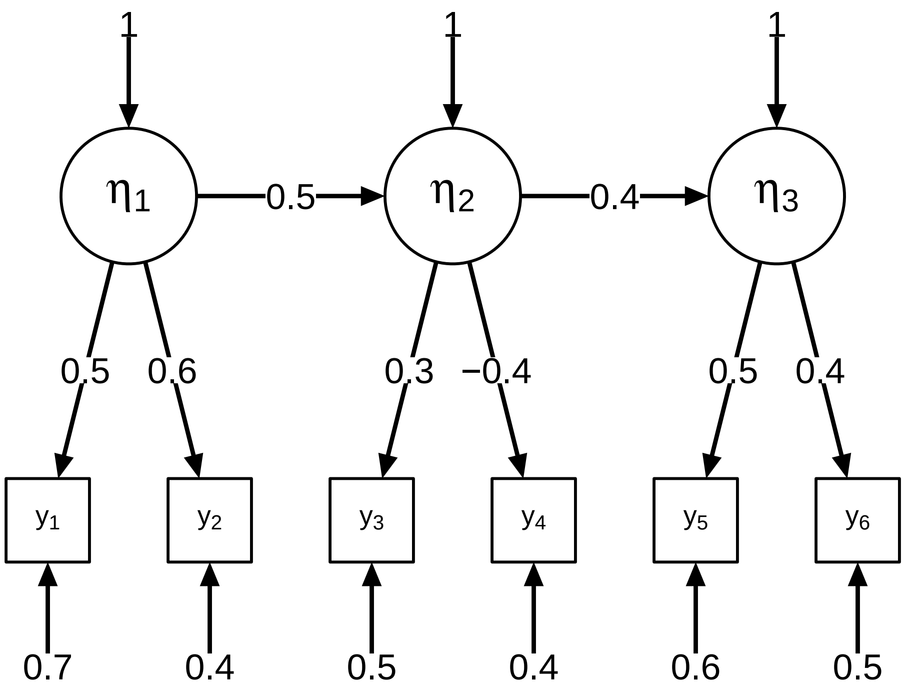
2. Covariance between White Similarity and Minority Connectedness
3. Variance of White Self-Categorization

Question 11 (2 points) Repeat the previous question, using Wright's rules instead of covariance algebra to obtain your answers. Confirm that your answers match the ones you got above

1. Covariance between Minority Appearance and Minority Self-Categorization
2. Covariance between White Similarity and Minority Connectedness
3. Variance of White Self-Categorization

Part 5. Modeling SEMs

Consider the following model:



Question 12 (1 point) Give the matrices $\mathbf{\Lambda}$, $\boldsymbol{B}$, $\mathbf{\Psi}$ and $\mathbf{\Theta}$ (containing values, not symbols).

Question 13 (1 point) Compute $\mathbf{\Sigma}$ using the all-y formula from the slides. Note that $\boldsymbol{I}$ is an identity matrix, a matrix with ones on the diagonal and zeroes elsewhere, of the same dimensions as $\boldsymbol{B}$.

It is known that $(\boldsymbol{I}-\boldsymbol{B})^{-1}$ can be written as:

$$\begin{aligned}(\boldsymbol{I}-\boldsymbol{B})^{-1} &= \boldsymbol{I}+\boldsymbol{B}+\boldsymbol{B}^2+\boldsymbol{B}^3+\boldsymbol{B}^4+\dots \\ &= \boldsymbol{I}+\boldsymbol{B}+\boldsymbol{BB}+\boldsymbol{BBB}+\boldsymbol{BBBB}+\dots\end{aligned} \tag{1}$$

if $\boldsymbol{B}$ is *nilpotent*, meaning that for some finite k, $\boldsymbol{B}^k = \boldsymbol{O}$ (a matrix containing only zeroes) and as a result for every $l \in \mathbb{Z}^+$ (l is a positive integer, such as 1, 2 or 3), $\boldsymbol{B}^{k+l} = \boldsymbol{O}$.

Question 14 (1 point) Show that $\boldsymbol{B}$ is nilpotent for $k \geq 3$ and compute $(\boldsymbol{I}-\boldsymbol{B})^{-1}$ using Equation (1). Also compute $(\boldsymbol{I}-\boldsymbol{B})^{-1}$ using R and compare your results. ■

Challenge question (1 bonus point) Prove Equation (1) ■