Assignment 3

SEM 1: Confirmatory Factor Analysis

Please hand in a `.pdf` file containing your report and a `.R` containing your codes or screenshots of every Jasp analysis. The deadline of this assignment is Sunday April 29 23:59.

Assignment

While still working at the University of Amsterdam, Wicherts and Dolan (2010) performed measurement invariance tests on data used by te Nijenhuis, Tolboom, Resing, and Bleichrodt (2004) that aimed to measure intelligence differences between ethnic majority and minority groups in the Netherlands. In particular, the following factor model was used:

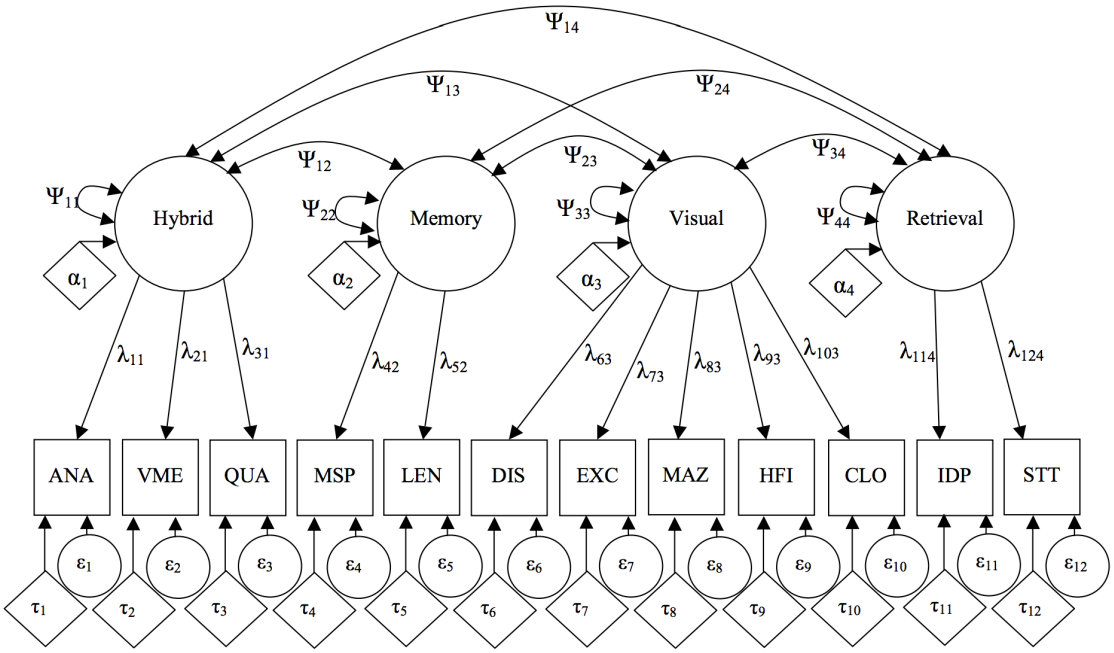

Table 2 of Wicherts and Dolan (2010) report the main results of the measurement invariance tests (note that the order of restrictions is slightly different than in the lecture):

Table 2

Fit measures of steps towards strict factorial invariance

Step	Restrictions	df	χ^2	Δdf	$\Delta\chi^2$	p	RMSEA	CFI	AIC
1	-	96	152.52**				.059	.962	318
2	**Factor loadings**	104	157.13**	8	4.61	.798	.055	.964	306
3	Factor loadings, **residual variances**	116	179.96**	12	22.83*	.029	.059	.957	310
3a	Factor loadings, residual variances[1]	115	170.93**	-1	9.03**	.003	.054	.962	299
4	Factor loadings, residual variances[1], **intercepts**	123	240.80**	8	69.87**	.000	.077	.920	356
4c	Factor loadings, residual variances[1], intercepts[2,3,4]	120	174.37**	-1	4.14*	.042	.052	.963	292

Note: Restrictions in bold are tested by loglikelihood test $\Delta\chi^2$. *p < 0.05; **p < 0.01; (-1): Parameter freely estimated;

1: Memory Span; 2: Verbal Meaning; 3: Learning Names; 4: Storytelling

Jelte has made papers and summary statistics needed to replicate the analyses available on his website. A free version of the paper can be found here (journal version is on blackboard) and the summary statistics needed for the analysis can be found here. A related paper you might find interesting to read is available here. We can read these into R by using the `WichertsData.R` file on blackboard. The sample size of the

majority group was 196 and the sample size of the minority group was 131. These variance–coviariance matrices and means can directly be used in a SEM analysis. To estimate a model using summary statistics, you can use the `sample.cov`, `sample.mean` and `sample.nobs` arguments. Note, multi-group analysis using summary statistics have not yet been implemented in Jasp. To this end, I have simulated the data `WichertsSimData_forJasp.csv` on blackboard that can be used in Jasp and should give about the same results. *Only* use this data if you use Jasp!

Question 1 (4 points) Replicate Table 2 of Wicherts and Dolan (2010) (disregarding steps 4a and 4b which are blanked out above). To obtain the right values, use the `mimic = "EQS"` option in lavaan (also available in Jasp). In Onyx, it is ok if you approximately get the same values. In Jasp, use the data file from blackboard, otherwise compute the model using summary statistics. ■

Question 2 (2 points) Write down the model matrices for model 3a, including subscripts denoting group for parameters that are free in both groups. Also list all parameters that are freely estimated (e.g., $\lambda_{12}, \lambda_{13}, \ldots$), and manually recompute the degrees of freedom (take restrictions into account!) ■

Question 3 (1 point) Consider the items in your group project. Could you think of one of the items that may feature intecept differences or factor loadings differences between two groups? Mention in your answer if you would expect intercept or factor loadings differences and why. The group does not have to be a group you measured. You may discuss this question with your group members, but write your answer alone. ■

Suppose I wish to estimte the following model for group 1:

$$\boldsymbol{\Lambda}_1 = \begin{bmatrix} \lambda_{111} & 0 \\ \lambda_{211} & 0 \\ \lambda_{311} & 0 \\ 0 & \lambda_{421} \\ 0 & \lambda_{521} \\ 0 & \lambda_{621} \end{bmatrix}, \boldsymbol{\Psi}_1 = \begin{bmatrix} 1 & \\ \psi_{211} & 1 \end{bmatrix}, \boldsymbol{\tau}_1 = \begin{bmatrix} \tau_{11} \\ \tau_{21} \\ \tau_{31} \\ \tau_{41} \\ \tau_{51} \\ \tau_{61} \end{bmatrix}, \boldsymbol{\alpha}_1 = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix}$$

$$\boldsymbol{\Theta}_1 = \begin{bmatrix} \theta_{111} & & & & & \\ & \theta_{221} & & & & \\ & & \theta_{331} & & & \\ & & & \theta_{441} & & \\ & & & & \theta_{551} & \\ & & & & & \theta_{661} \end{bmatrix}$$

and for group 2:

$$\boldsymbol{\Lambda}_2 = \begin{bmatrix} \lambda_{112} & 0 \\ \lambda_{212} & 0 \\ \lambda_{312} & 0 \\ \lambda_{412} & \lambda_{422} \\ 0 & \lambda_{522} \\ 0 & \lambda_{622} \end{bmatrix}, \boldsymbol{\Psi}_2 = \begin{bmatrix} 1 & \\ \psi_{212} & 1 \end{bmatrix}, \boldsymbol{\tau}_2 = \begin{bmatrix} \tau_{12} \\ \tau_{22} \\ \tau_{32} \\ \tau_{42} \\ \tau_{52} \\ \tau_{62} \end{bmatrix}, \boldsymbol{\alpha}_2 = \begin{bmatrix} \alpha_{12} \\ \alpha_{22} \end{bmatrix}$$

$$\boldsymbol{\Theta}_2 = \begin{bmatrix} \theta_{112} & & & & & \\ & \theta_{222} & & & & \\ & & \theta_{332} & & & \\ & & & \theta_{442} & & \\ & & & & \theta_{552} & \\ & & & & & \theta_{662} \end{bmatrix}$$

Question 4 (1 point) Calculate the degrees of freedom. ■

Question 5 (1 point) Is this model identified? Explain your answer. ■

Question 6 (1 point) If this model fits (after potentially identifying the model), would that indicate that *configural invariance* holds? ■

Question 7 (2 points) Are the following statements true or false?

- When comparing two models, the *simpler* model is the model in which there are *more* degrees of freedom.
- If a χ^2 difference test between two nested models is *significant*, we would prefer to select the *simpler* model.
- If the AIC is lower for model A than for model B, then so is the BIC.
- We would always prefer the model with the lowest *RMSEA*.

■

Question 8 (1 points) Compute the required sample size to reject H_0:

$$H_0 : \text{RMSEA} < 0.05$$

given that we assume RMSEA $= 0.08$ with 25 degrees of freedom. Do the same for the hypothesis:

$$H_0 : \text{RMSEA} > 0.08$$

assuming RMSEA $= 0.01$ with 25 degrees of freedom. How are these hypotheses called? ■

References

te Nijenhuis, J., Tolboom, E., Resing, W., & Bleichrodt, N. (2004). Does cultural background influence the intellectual performance of children from immigrant groups? *European Journal of Psychological Assessment*, *20*(1), 10–26.

Wicherts, J. M., & Dolan, C. V. (2010). Measurement invariance in confirmatory factor analysis: An illustration using iq test performance of minorities. *Educational Measurement: Issues and Practice*, *29*(3), 39–47.