Psychological Networks Summer School

Day 4, part 2: Discovering Psychological Dynamics in Time-series Data

Sacha Epskamp

07-07-2016
Goal

- Up to *three* network structures can be obtained in time-series data:
  - Contemporaneous networks
  - Temporal networks
  - Between-subjects networks

- All three structures can be interpreted in several ways:
  - Highlighting potential causal pathways
  - Showing predictive effects and mediation
  - Under very strict assumptions: a causal model
Outline

- When cases are independent
  - Cross-sectional data
  - The Gaussian Graphical model
  - Interpreting network structures
- When cases are not independent: \( N = 1 \)
  - The VAR model
  - Temporal and contemporaneous networks and causation
- When cases are not independent: \( N > 1 \)
  - The multi-level VAR model
  - Between-subjects networks and causation
- Conclusion
Experience Sampling Method

- Rows are termed the cases
Cross-sectional Data
Cross-sectional Data

- Every person measured only once
- Cases can reasonably be assumed to be independent
  - Given IQ has a mean of 100 and SD of 15, does knowing that Peter has an IQ of 90 help us predict better that Sarah had an IQ of 110?
- Because of this assumption, likelihood reduces to a product
  - $Y \sim N(\mu, \Sigma)$
  - $f(\mathbf{y} | \mu, \Sigma) = \prod_{p=1}^{N} f(\mathbf{y}^{(p)} | \mu, \Sigma)$
The Gaussian Graphical Model

- $\Sigma$, the variance-covariance matrix, encodes all information how variables relate to one-another.
- Because of the Schur complement, it also encodes all conditional relationships.
- We will focus on its inverse, $K$:
  - $K = \Sigma^{-1}$
- The inverse variance-covariance matrix is called a Gaussian graphical model (GGM):
  - Encodes an undirected network.
- GGM is a network of partial correlation coefficients:
  - \( \text{Cor} \left( Y_i, Y_j \mid Y^{-(i,j)} \right) = -\frac{\kappa_{ij}}{\sqrt{\kappa_{ii}\kappa_{jj}}} \)
The GGM model:

- Concentration – Fatigue – Insomnia

Is equivalent to three causal structures:

1. Concentration → Fatigue → Insomnia
2. Concentration ← Fatigue → Insomnia
3. Concentration ← Fatigue ← Insomnia

Thus, the GGM highlights potential causal pathways
GGM and Multiple Regressions

Y_1

Y_2

Y_3

Y_4
GGM and Multiple Regressions

\[ y_1 = \tau_1 + \gamma_{12}y_2 + \gamma_{13}y_3 + \gamma_{14}y_4 + \varepsilon_1 \]
GGM and Multiple Regressions

\[ y_2 = \tau_2 + \gamma_{21}y_1 + \gamma_{23}y_3 + \gamma_{24}y_4 + \varepsilon_2 \]
GGM and Multiple Regressions

\[ y_3 = \tau_3 + \gamma_{31}y_1 + \gamma_{32}y_2 + \gamma_{34}y_4 + \varepsilon_3 \]
GGM and Multiple Regressions

\[ y_4 = \tau_4 + \gamma_{41} y_1 + \gamma_{42} y_2 + \gamma_{43} y_3 + \varepsilon_4 \]
GGM and Multiple Regressions

Y_4

Y_1

Y_2

Y_3

\gamma_{41}, \gamma_{14}, \gamma_{12}, \gamma_{21}, \gamma_{32}, \gamma_{23}, \gamma_{34}, \gamma_{43}
GGM and Multiple Regressions

\[ \rho_{ij} = \frac{\gamma_{ij} \text{Var}(\varepsilon_j)}{\text{Var}(\varepsilon_i)} = \frac{\gamma_{ji} \text{Var}(\varepsilon_i)}{\text{Var}(\varepsilon_j)} \]
- LASSO model selection using Extended Bayesian information criterion (EBIC)
Emperical Example: Personality

I will analyze the BFI dataset from the psych package:

- 25 items
- 2800 subjects
- Five items for each of the five central personality traits

```r
library("psych")
?bfi
```
Partial Correlation Network

- **A**: Am indifferent to the feelings of others.
- **A2**: Inquire about others’ well-being.
- **A3**: Know how to comfort others.
- **A4**: Love children.
- **A5**: Make people feel at ease.

- **C**: Am exacting in my work.
- **C2**: Continue until everything is perfect.
- **C3**: Do things according to a plan.
- **C4**: Do things in a half-way manner.
- **C5**: Waste my time.

- **E**: Don’t talk a lot.
- **E2**: Find it difficult to approach others.
- **E3**: Know how to captivate people.
- **E4**: Make friends easily.
- **E5**: Take charge.

- **N**: Get angry easily.
- **N2**: Get irritated easily.
- **N3**: Have frequent mood swings.
- **N4**: Often feel blue.
- **N5**: Panic easily.

- **O**: Am full of ideas.
- **O2**: Avoid difficult reading material.
- **O3**: Carry the conversation to a higher level.
- **O4**: Spend time reflecting on things.
- **O5**: Will not probe deeply into a subject.
Cross-sectional Data
Time-series: $n = 1$
Time-series: $n > 1$

Conclusion

A1: Am indifferent to the feelings of others.
A2: Inquire about others' well-being.
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C1: Am exacting in my work.
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E1: Don't talk a lot.
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O1: Am full of ideas.
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glasso network
<table>
<thead>
<tr>
<th>Cross-sectional Data</th>
<th>Time-series: $n = 1$</th>
<th>Time-series: $n &gt; 1$</th>
<th>Conclusion</th>
</tr>
</thead>
</table>

**Time-series: $n = 1$**
Time-series Data

- One person measured several times in a short period
- Cases can **not** reasonably be assumed to be *independent*
  - Knowing someone’s level of fatigue at a time point helps predict his or her level of fatigue at the next time point.
- Likelihood not easy to compute without three assumptions:
  - The time-series factorize according to a graph
  - The model does not change over time
  - The first measurement is exogenous
- We will use the lag-1 factorization
Vector Auto-regression

\[ Y_t \mid y_{t-1} \sim N(\mu + B(y_{t-1} - \mu), \Theta) \]

- \( B \) encodes the *temporal network*
  - Granger causality
- \( \Theta^{-1} \) encodes the *contemporaneous network*
  - GGM
- The sample means can be used as plugin to center the predictors
Cross-sectional Data

Time-series: $n = 1$

Time-series: $n > 1$

Conclusion

Temporal network

- Exercising → Energetic: $0.25$

Contemporaneous network

- Exercising → Energetic: $0.3$
- Energetic → Exercising: $0.1$
Contemporaneous Causation

- Many causal effects likely faster than the time-window of measurement
  - Somatic arousal $\rightarrow$ anticipation of panic attack $\rightarrow$ anxiety
- These can be caught in a contemporaneous network of partial correlations
- Thus, the contemporaneous network can also be seen to highlight potential causal relationships
- As the contemporaneous network is the GGM, the temporal network can be seen as a correction for dependent measurements in estimating the GGM
- Estimation straightforward using multiple regression
- For model selection, we use the graphical VAR model
- Estimation via LASSO regularization, using EBIC to select optimal tuning parameter
- We implemented these methods in the R package *graphicalVAR*
- Also implemented in *sparseTSCGM*
Empirical Example

Data collected by Date C. Van der Veen, in collaboration with Harriette Riese en Renske Kroeze.

- Patient suffering from panic disorder and depressive symptoms
  - Perfectionist
- Measured over a period of two weeks
- Five times per day
- Items were chosen after intake together with therapist
Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion
1: I feel anxious
2: I feel stressed
3: I feel angry
4: I feel sad
5: I feel guilty
6: I feel weak
7: I feel worthless
8: I feel helpless
9: I feel full of Energy
10: I am afraid of a panic attack
11: I am afraid I am going to cry
12: I am afraid of appearing angry
13: I have 'had to do things'
14: I am experiencing bodily discomfort
15: I am enjoying myself
16: I let something pass I find important
17: I experienced my social environment as pleasurable
18: I was physically active
Feeling worthless interacts with feeling helpless
Feeling stressed interacts with feeling the need to do things
Central node: Feeling sad

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Cycle of enjoyment, feeling sad, feeling worthless and being active
Having to had to do things leads to letting important things pass.

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Fear of panic attack is not connected
Time-series: $n > 1$
A Network Approach to Psychopathology: New Insights into Clinical Longitudinal Data

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1Department of Psychology, University of Leuven, Leuven, Belgium, 2Department of Psychiatry and Neuropsychology, Maastricht University, Maastricht, The Netherlands, 3Department of Clinical Psychological Science, Maastricht University, Maastricht, The Netherlands, 4Department of Psychology, University of Amsterdam, Amsterdam, The Netherlands

Abstract

In the network approach to psychopathology, disorders are conceptualized as networks of mutually interacting symptoms (e.g., depressed mood) and transdiagnostic factors (e.g., rumination). This suggests that it is necessary to study how symptoms dynamically interact over time in a network architecture. In the present paper, we show how such an architecture can be constructed on the basis of time-series data obtained through Experience Sampling Methodology (ESM). The proposed methodology determines the parameters for the interaction between nodes in the network by estimating a multilevel vector autoregression (VAR) model on the data. The methodology allows combining between-subject and within-subject information in a multilevel framework. The resulting network architecture can subsequently be analyzed through network analysis techniques. In the present study, we apply the method to a set of items that assess mood-related factors. We show that the analysis generates a plausible and replicable network architecture, the structure of which is related to variables such as neuroticism; that is, for subjects who score high on neuroticism, worrying plays a more central role in the network. Implications and extensions of the methodology are discussed.

Multi-level VAR

- Each subject is assumed to have their own temporal and contemporaneous VAR model
- VAR parameters come from distribution
  - Fixed effect
  - Random effect
Multi-level VAR

Adding superscript \( p \) for subject. Level 1 model:

\[
Y_{t}^{(p)} | y_{t}^{(p)} = N \left( \mu^{(p)} + B^{(p)}y_{t-1}^{(p)}, \Theta^{(p)} \right)
\]

Level 2 model:

\[
\begin{bmatrix}
\mu^{(p)} \\
\text{Vec} \left( B^{(p)} \right)
\end{bmatrix}
\sim N \left( f, \Omega \right).
\]

\( f \) encodes fixed effects and \( \Omega \) the distribution of random effects.
Cross-sectional Data

Time-series: $n = 1$

Time-series: $n > 1$

Conclusion

$Y_1$

$Y_2$

$\beta_{11}$

$\beta_{12}$

$\beta_{21}$

$\beta_{22}$

$\mu_1$

$\mu_2$

1

1
Each Parameter has a Distribution
Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion

### Individual Networks

**Bob**

- \( Y_1 \):
  - 0.15
  - 0.15
  - 0.16
  - 0.2
  - 0.28
  - -0.58

- \( Y_2 \):
  - 0.15
  - 0.15
  - 0.2
  - 0.29
  - -0.54

**Alice**

- \( Y_1 \):
  - 0.06
  - 0.07
  - -0.1
  - 0.22
  - 0.29

- \( Y_2 \):
  - 0.06
  - 0.07
  - -0.1
  - 0.22
  - 0.29
Random Effects

Cross-sectional Data

Time-series: $n = 1$

Time-series: $n > 1$

Conclusion
Fixed Effects

Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion
Fixed Effects

\[ Y_1 \] → \[-0.6\] \[ Y_2 \] → \[ 0.07 \]

\[ Y_1 \] \[ 1 \] \[ 0.18 \] \[ Y_2 \] \[ 1 \] \[ 0.34 \]

Cross-sectional Data

\[ \text{Time-series: } n = 1 \]

\[ \text{Time-series: } n > 1 \]

Conclusion
Individiual Differences
### Parameter correlation Matrix

![Parameter correlation Matrix](image)
## Parameter Correlation Matrix

<table>
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<tr>
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<th>Y₂</th>
<th>Y₃</th>
<th>Y₄</th>
<th>Y₅</th>
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## Stability

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### Connectivity

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Emotion-Network Density in Major Depressive Disorder

Madeline Lee Pe¹, Katharina Kircanski², Renee J. Thompson³, Laura F. Bringmann¹, Francis Tuerlinckx¹, Merijn Mestdagh¹, Jutta Mata⁴, Susanne M. Jaeggi⁵, Martin Buschkuehl⁶, John Jonides⁷, Peter Kuppens¹, and Ian H. Gotlib²

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# Between-subject Effects

<table>
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Between-subjects Network

The random effects variance-covariance matrix can be divided into four blocks:

\[
\begin{bmatrix}
R_{\mu} \\
R_{B}
\end{bmatrix} \sim N \left(0, \begin{bmatrix}
\Omega_{\mu} & \Omega_{\mu B} \\
\Omega_{B\mu} & \Omega_{B}
\end{bmatrix}\right).
\]

- Block $\Omega_{\mu}$ encodes the between-subject relationships between means.
- These can be used to estimate a GGM.
  - Between-subjects network of partial correlations.
Hypothetical example of networks based on two persons:

- Clinically depressed person constantly scoring high on both
- Healthy person constantly scoring low on both
**Temporal Estimation**

- Multi-variate multi-level MLE regression estimation is complicated and not yet well implemented in open source software
- `lme4` packages implements univariate multi-level regression
- `lmer` function

- A multi-level VAR model can be estimated by sequentially estimating univariate models
  - Estimate all incoming edges per node
Temporal Estimation

- **Correlated estimation:**
  - Needs to integrate out a high-dimensional distribution over parameters
  - Only feasible for up to \( \sim 6 \) nodes
  - Does not estimate all parameter covariances
    - Not all parameters together in the same model

- **Orthogonal estimation**
  - Alternatively, parameter covariances can be fixed to zero
  - Fast, and works for high dimensions (e.g., 20 nodes)
  - But, does not return any parameter correlation
Correlated Estimation

Cross-sectional Data

Time-series: $n = 1$

Time-series: $n > 1$

Conclusion
Orthogonal Estimation

Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion
Between-subject Estimation

- Between subject effects can be obtained by centering predictors and adding the person-means as level 2 predictors
- This can be seen as node-wise estimation of a GGM
- Thus, an estimate for the between-subjects GGM can be obtained by averaging the level-2 predictive effects standardized with the residual variances
Contemporaneous Estimation

- Contemporaneous networks need to be estimated post-hoc by investigating the residuals.
- Either inverting the sample variance-covariance matrix of residuals:
  - Fixed
  - Unique
- Or as a second multi-level model using nodewise estimation of a GGM:
  - Correlated
  - Orthogonal
Simulation Studies

- 8 and 20 nodes
- Random between-subjects covariance matrix for means and temporal effects
  - `clusterGeneration` R package with “onion” method
  - No correlations between means and temporal effects
- Temporal effects scaled to enforce stationarity
- Random fixed contemporaneous covariance matrix
- Contemporaneous person-specific covariances drawn from Wishart distribution with $2P$ DF
- Performance checked with temporal effects and partial correlations
- Each condition (# persons, # time, temporal estimation method and contemporaneous estimation method) replicated 100 times
Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion

Contemporaneous: correlated
Contemporaneous: fixed
Contemporaneous: orthogonal
Contemporaneous: unique

Temporal network fixed effects – 8 nodes

# of subjects
Correlation
Measurements per subject
25
50
75
100
Cross-sectional Data

Time-series: $n = 1$

Time-series: $n > 1$

Conclusion

Contemporaneous: correlated
Contemporaneous: fixed
Contemporaneous: orthogonal
Contemporaneous: unique

Temporal: correlated
Temporal: fixed
Temporal: orthogonal
Temporal: unique

# of subjects
Correlation
Measurements
per subject
25
50
75
100

Temporal network random effects – 8 nodes
Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion

Contemporaneous: correlated
Contemporaneous: fixed
Contemporaneous: orthogonal
Contemporaneous: unique

Temporal: correlated
Temporal: fixed
Temporal: orthogonal
Temporal: unique

Measurements per subject

- 25
- 50
- 75
- 100
Cross-sectional Data

Time-series: \( n = 1 \)

\( n > 1 \)

Conclusion

Contemporaneous network random effects – 8 nodes

Correlation Measurements per subject

Temporally correlated Temporally fixed Temporally orthogonal Temporally unique

# of subjects

50 100 250 500 50 100 250 500 50 100 250 500
Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion

Contemporaneous: correlated

Contemporaneous: fixed

Contemporaneous: orthogonal

Contemporaneous: unique

Between-subjects network – 8 nodes

Correlation

Measurements per subject

# of subjects

Temporal: correlated

Temporal: fixed

Temporal: orthogonal

Temporal: unique

# of subjects

Between-subjects network – 8 nodes

Correlation

Measurements per subject

# of subjects
Cross-sectional Data

Time-series: $n = 1$

Time-series: $n > 1$

Conclusion

Contemporaneous: fixed
Contemporaneous: orthogonal
Contemporaneous: unique

Temporal network fixed effects – 20 nodes

Correlation

Measurements per subject

# of subjects
Cross-sectional Data

- Time-series: $n = 1$
- Time-series: $n > 1$

Conclusion

Contemporaneous: fixed
Contemporaneous: orthogonal
Contemporaneous: unique

Temporal network random effects – 20 nodes

<table>
<thead>
<tr>
<th>Correlation</th>
<th># of subjects</th>
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<tbody>
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<td>50</td>
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<tr>
<td>0.5</td>
<td>100</td>
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<tr>
<td>0.0</td>
<td>250</td>
</tr>
<tr>
<td>-0.5</td>
<td>500</td>
</tr>
</tbody>
</table>

Measurements per subject:
- 25
- 50
- 75
- 100

Temporal: fixed
Temporal: orthogonal
Temporal: unique
Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion

Contemporaneous: fixed

Contemporaneous: orthogonal

Contemporaneous: unique

Temporal: fixed

Temporal: orthogonal

Temporal: unique

Correlation Measurements per subject

25

50

75

100

Contemporaneous network fixed effects – 20 nodes

# of subjects
Cross-sectional Data  Time-series: $n = 1$

Time-series: $n > 1$

Contemporaneous network random effects – 20 nodes

<table>
<thead>
<tr>
<th>Contemporaneous: fixed</th>
<th>Contemporaneous: orthogonal</th>
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</table>

- Time-series: $n = 1$
  - Cross-sectional Data
  - Time-series: $n = 1$
- Time-series: $n > 1$
  - Contemporaneous: fixed
  - Contemporaneous: orthogonal
  - Contemporaneous: unique

Measurements per subject:
- 25
- 50
- 75
- 100

# of subjects

Conclusion
Cross-sectional Data

Time-series: n = 1

Time-series: n > 1

Conclusion

Contemporaneous: fixed
Contemporaneous: orthogonal
Contemporaneous: unique

Temporal: fixed
Temporal: orthogonal
Temporal: unique

Correlation Measurements per subject
25
50
75
100

Between-subjects network – 20 nodes

# of subjects
Empirical Example

- Two datasets
  - Original: 26 subjects, 51 measurements on average, 1323 total observations
  - Replication: 65 subjects, 35.5 measurements on average, 2309 total observations
- 16 indicators of neuroticism, extroversion, conscientiousness
- Orthogonal estimation of temporal and contemporaneous effects
- Only significant effects shown
  - Alpha = 0.05 and using the “or” rule
- Very preliminary results
Cross-sectional Data

Time-series: \( n = 1 \)

Time-series: \( n > 1 \)

Conclusion

Between-subjects

- Worried
- Organized
- Ambitious
- Depressed
- Outgoing
- Selfconscious
- Selfdisciplined
- Energetic
- Frustrated
- Focused
- Guilty
- Adventurous
- Happy
- Control
- Achieved
- Angry
- Depressed
- Outgoing
- Ambitious
- Energetic
Conclusion
Conclusion

- Network structures are useful in discovering potential causal relationships
- Cross-sectional data:
  - Gaussian graphical model (GGM)
- Time-series data:
  - Contemporaneous network (GGM)
  - Temporal network (VAR)
  - Between-subjects network (GGM)
Limitations and Future Directions

- A lot of potential problems with multi-level estimation
  - Multivariate estimation
  - Modeling random contemporaneous effects
  - Parameter variance-covariances
  - Model selection

- Possibly move away from multi-level
  - LASSO variants?

- Lag-interval
The Limit of Observational Data

- Network structures are only hypothesis generating
  - Highlighting potential causal pathways
- Observational data can *never* confirm causality
  - Mixture of experimental and observational data needed
- We need to completely rethink the modeling framework to do so
Thank you for your attention!