Discovering Psychological Dynamics

The Gaussian Graphical Model in Cross-sectional and Time-series data

Sacha Epskamp

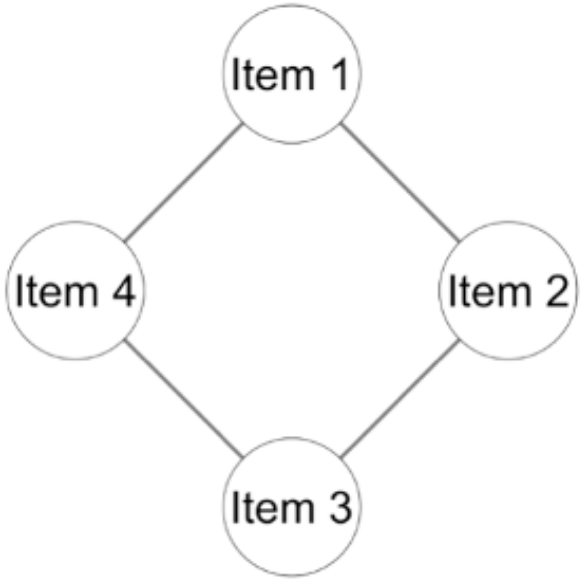
28-11-2016

Outline

- When cases are independent
  - Cross-sectional data
  - The Gaussian Graphical model
  - Interpreting network structures
- When cases are not independent: $N = 1$
  - The VAR model
  - Temporal and contemporaneous networks and causation
- When cases are not independent: $N > 1$
  - The multi-level VAR model
  - Between-subjects networks and causation
- Conclusion

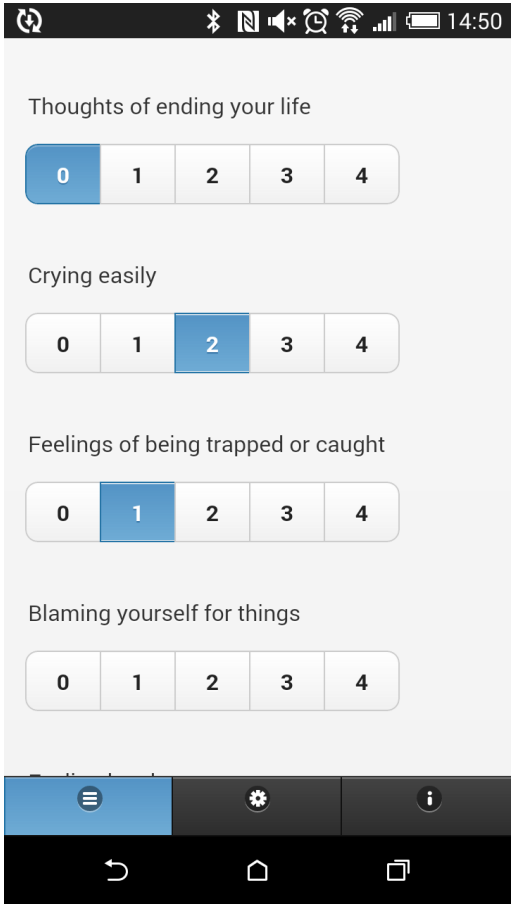
Network Psychometrics

	Item 1	Item 2	Item 3	Item 4
Subject 1	2	2	1	3
Subject 2	3	2	3	2
Subject 3	2	1	2	4
Subject 4	5	5	5	5
Subject 5	3	3	2	2
Subject 6	5	5	3	3
Subject 7	4	5	5	4
Subject 8	5	5	5	5
Subject 9	2	5	4	4
Subject 10	5	4	5	4



- What is the structure of psychology?
- *Psychological Networks*

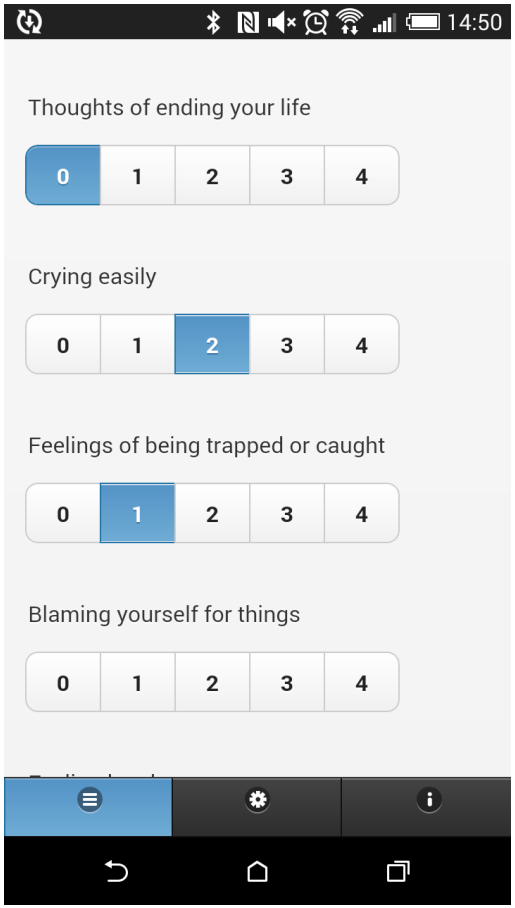
Psychological Data



Subject	Time	Item 1	Item 2	Item 3
Subject 1	Time 1	2	2	4
Subject 2	Time 1	4	1	2
Subject 3	Time 1	1	3	0

- Multiple people measured once: *cross-sectional analysis*

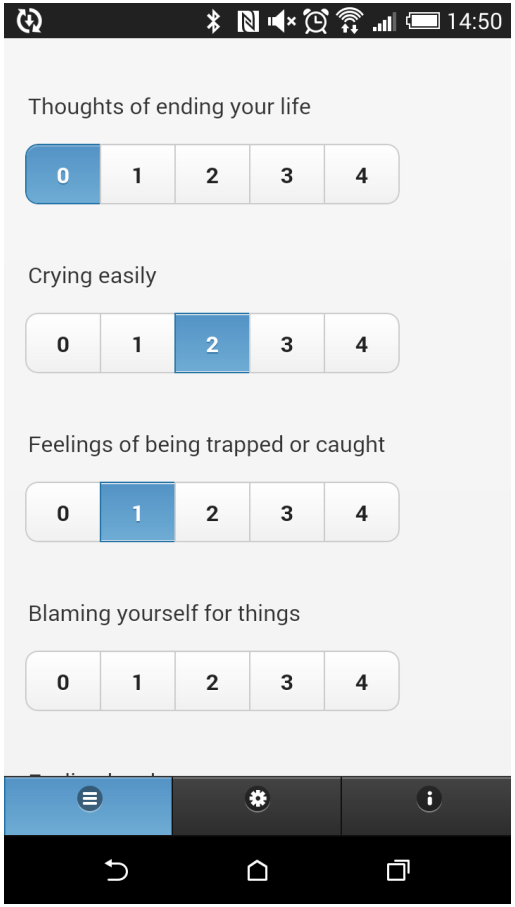
Psychological Data



Subject	Time	Item 1	Item 2	Item 3
Subject 1	Time 1	2	2	4
Subject 1	Time 2	2	4	4
Subject 1	Time 3	1	4	3

- Multiple people measured once: *cross-sectional analysis*
- One person measured multiple times: $N = 1$ *time-series*

Psychological Data



Subject	Time	Item 1	Item 2	Item 3
Subject 1	Time 1	2	2	4
Subject 1	Time 2	2	4	4
Subject 1	Time 3	1	4	3
Subject 2	Time 1	4	1	2
Subject 2	Time 2	3	1	2
Subject 2	Time 3	3	2	1
Subject 3	Time 1	1	3	0
Subject 3	Time 2	4	0	1
Subject 3	Time 3	0	3	3

- Multiple people measured once: *cross-sectional analysis*
- One person measured multiple times: $N = 1$ *time-series*
- Multiple people measured multiple times: $N > 1$ *time-series*

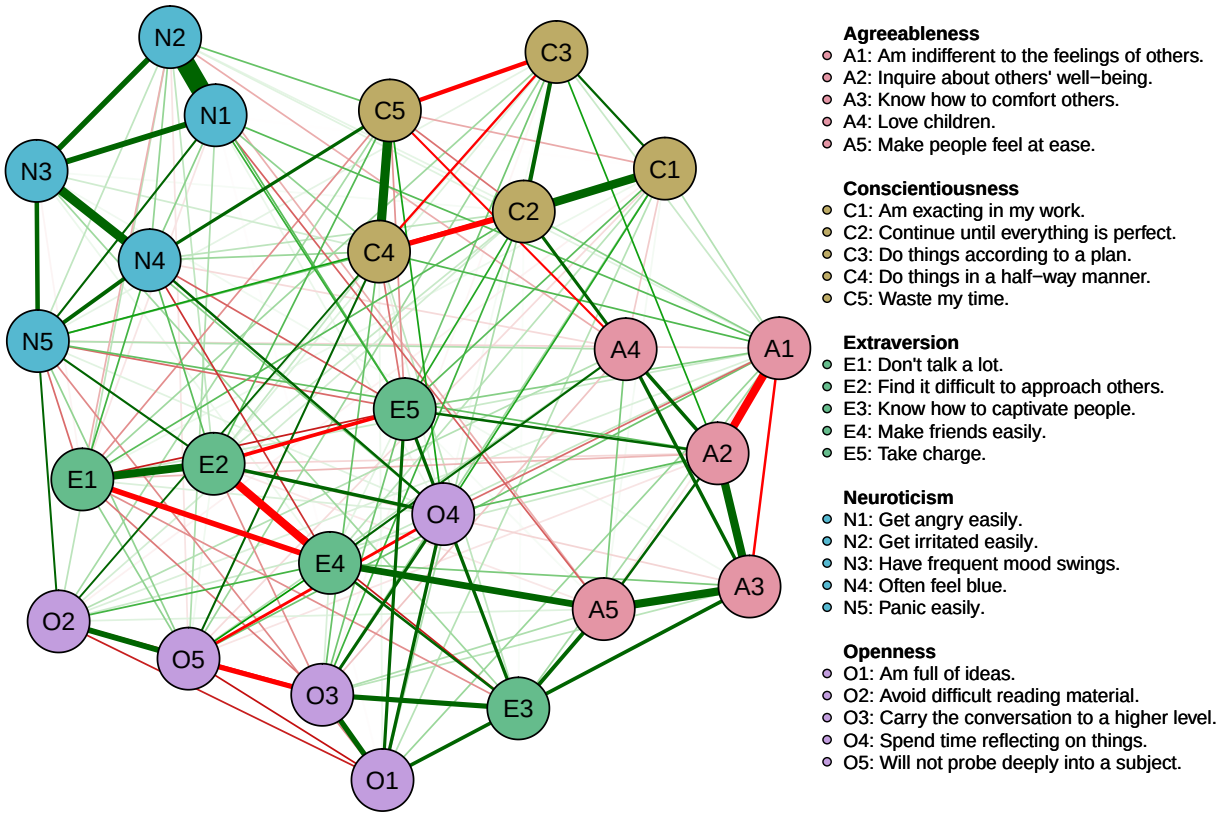
Three Types of Psychological Networks

Network	Gaussian	Binary	Mixed
Concentration	GGM	Ising	MGM
Contemporaneous	GGM	?	?
Temporal	VAR	Logistic VAR	MGM
Between-subjects	GGM	?	?

GGM = Gaussian graphical model; VAR = Vector Autoregression; MGM = Mixed graphical models

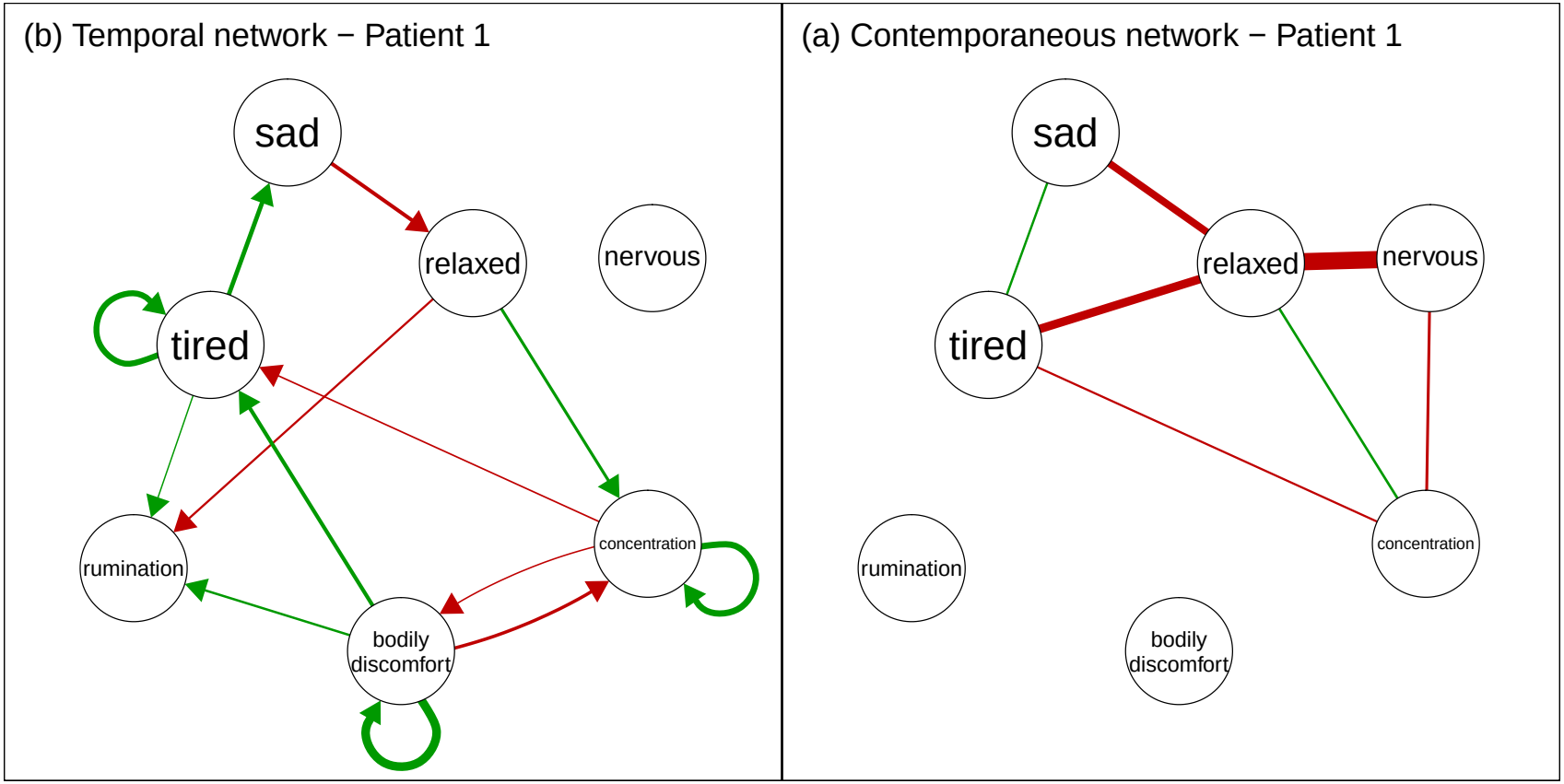
- Cross-sectional: concentration network
- $N = 1$ time-series: contemporaneous and temporal networks
  - The concentration network is the same as the contemporaneous network in time-series analysis
- $N > 1$ time-series: contemporaneous, temporal and between-subject networks

Cross-sectional Analysis



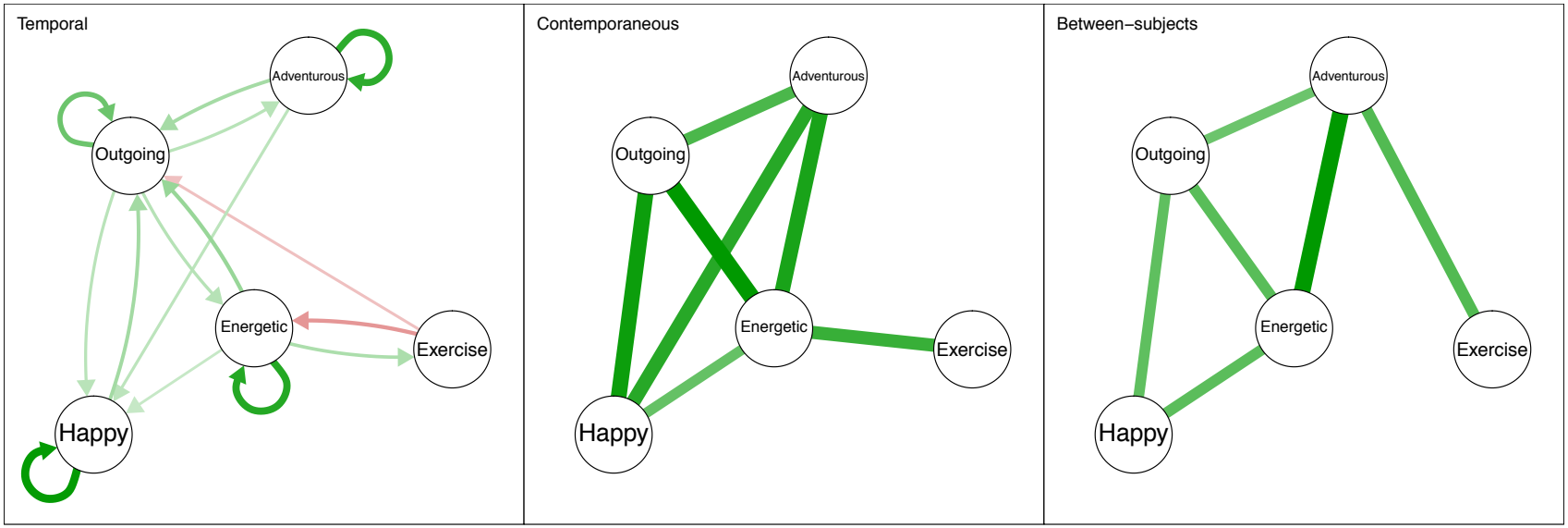
- Concentration network: unique variance between two variables

$N = 1$ Time-series Analysis



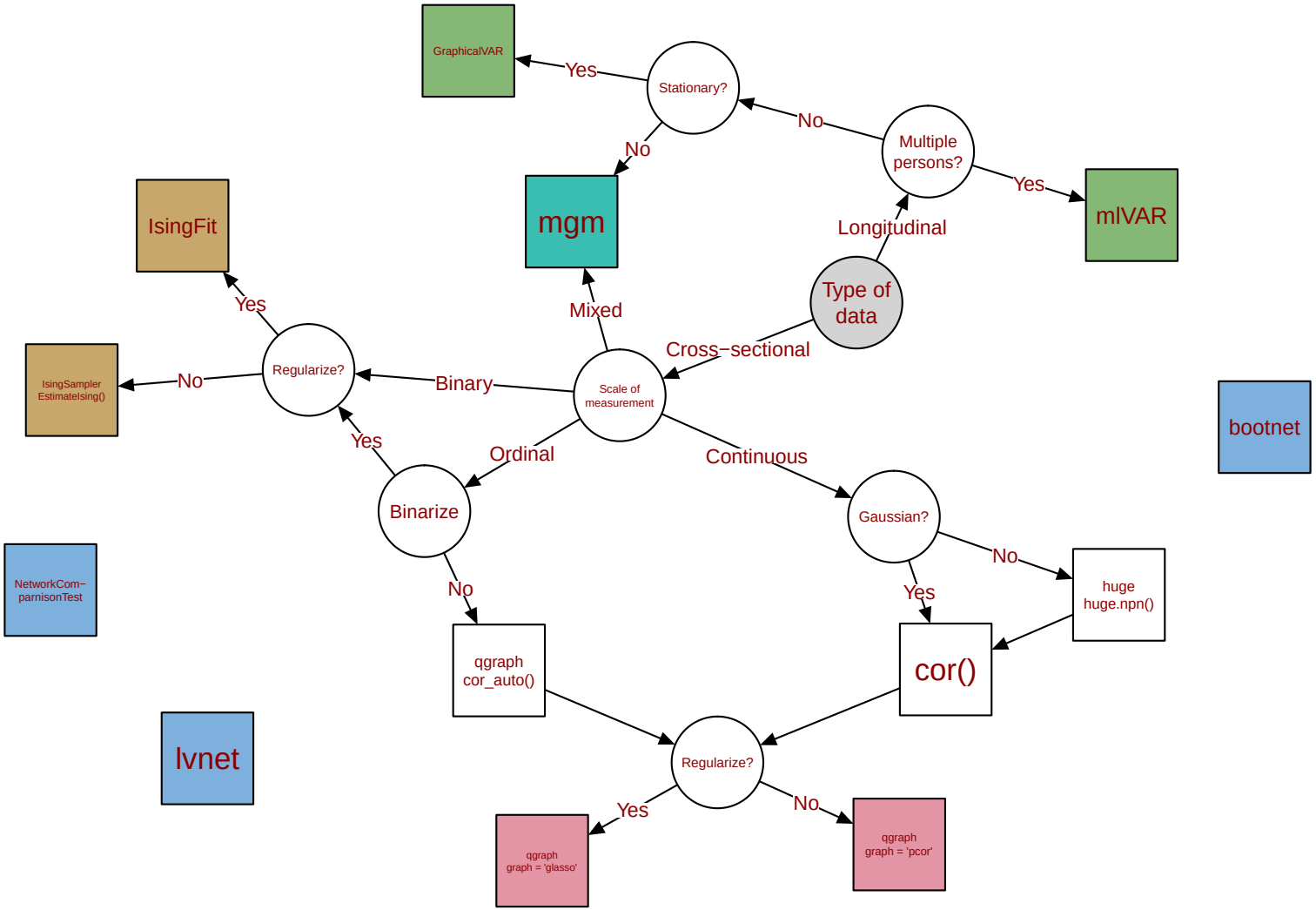
- Contemporaneous network: conditional concentration given $t - 1$
- Temporal network: regression coefficients between $t - 1$ and t

$N > 1$ Time-series Analysis



- Between-subjects network: concentration network between stationary means
- Two-step multilevel VAR

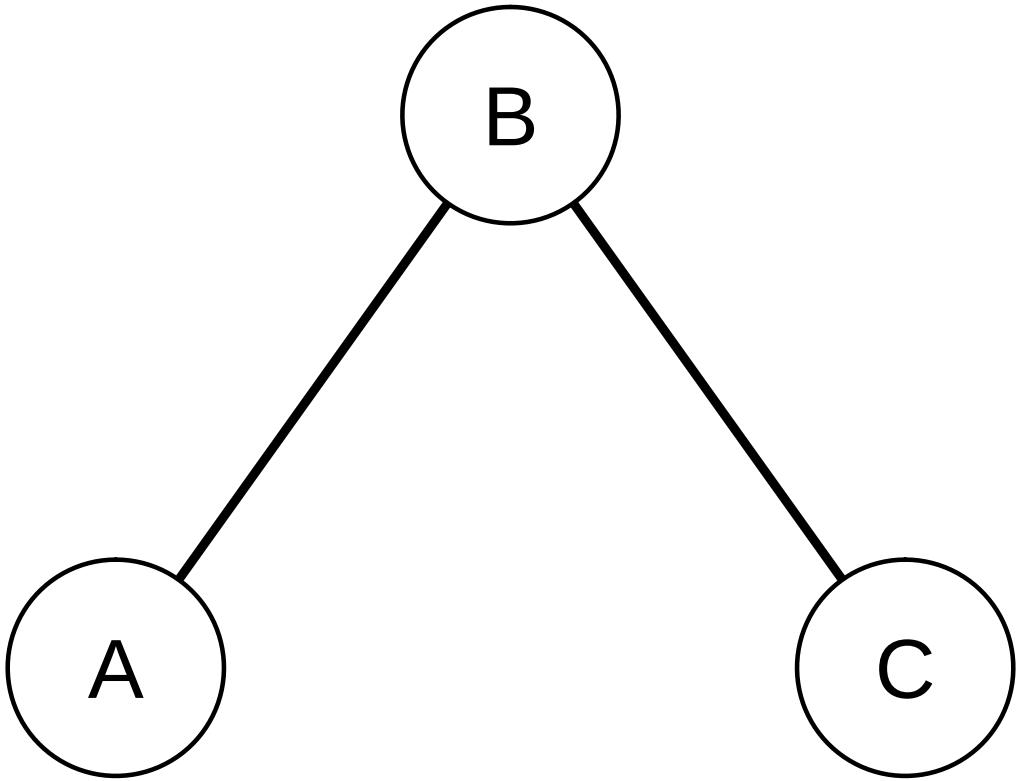
The Psychosystems Ecosystem



Concentration Networks

Concentration Networks

- A Concentration network is an undirected network
  - Termed a pairwise Markov Random Field (MRF)
- Two nodes are connected if they are not independent conditional on all other nodes.
- More importantly, two nodes are NOT connected if they are independent conditioned on all nodes:
- $X_i \perp\!\!\!\perp X_j \mid \boldsymbol{X}^{-(i,j)} = \boldsymbol{x}^{-(i,j)} \iff (i,j) \notin E$



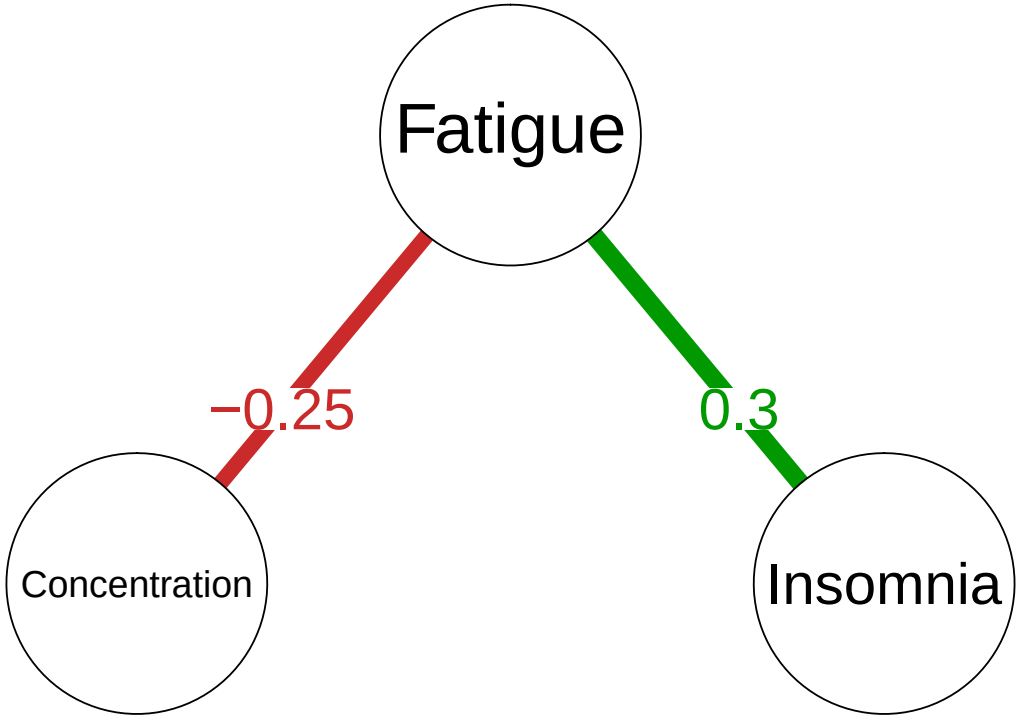
- B separates A and C
- $A \perp\!\!\!\perp C \mid B$

Markov Random Fields in Other Sciences

- MRF's have been used in many disciplines in Science
  - Physics
  - Machine Learning
  - Artificial Intelligence
  - Biometrics
  - Economics
  - Image processing
  - Neural Networks

Different kinds of Concentration Networks

- Multivariate normal data: Gaussian graphical models (partial correlation networks)
  - `https://arxiv.org/abs/1609.04156`
- Binary data: Ising model
  - `http://www.nature.com/articles/srep05918`
- Mixed data: Mixed graphical models
  - `https://arxiv.org/abs/1510.05677`



- Edge-weights encode conditional association between two variables after conditioning on all other variables in the network
- In the multivariate normal case: partial correlation coefficients

Interpreting a Concentration Network

The edges in a concentration network can be interpreted in several ways:

- Predictive effects
- A representation of conditional independence relationships
- Pairwise interactions
- Genuine symmetric relationships between nodes
  - Ising Model

The MRF model:

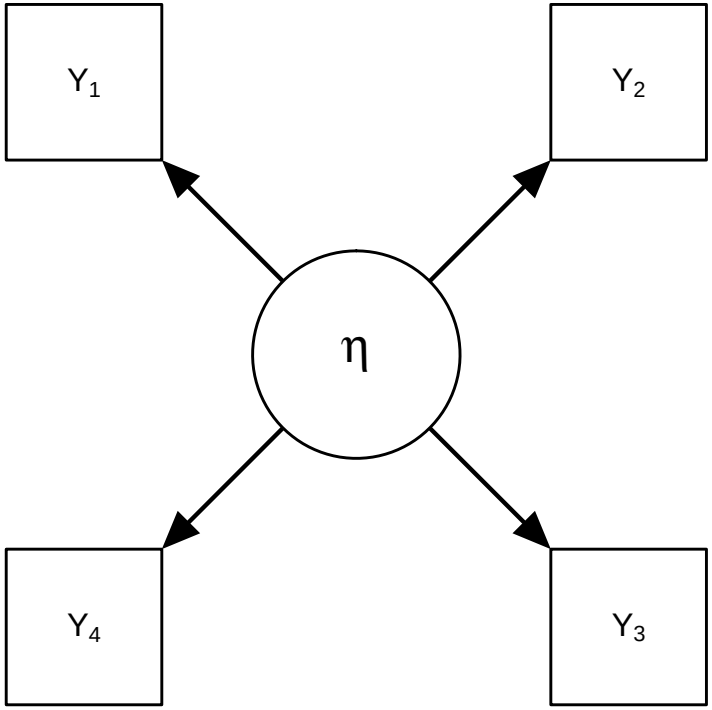
- Concentration – Fatigue – Insomnia

Is equivalent to three causal structures:

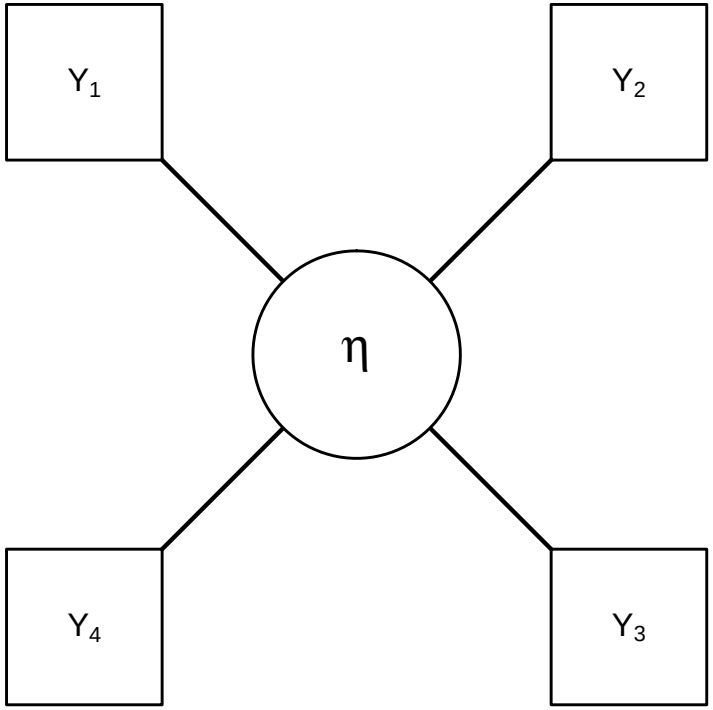
1. Concentration $\rightarrow$ Fatigue $\rightarrow$ Insomnia
2. Concentration $\leftarrow$ Fatigue $\rightarrow$ Insomnia
3. Concentration $\leftarrow$ Fatigue $\leftarrow$ Insomnia

Thus, MRF highlights potential causal pathways

Data generating structure

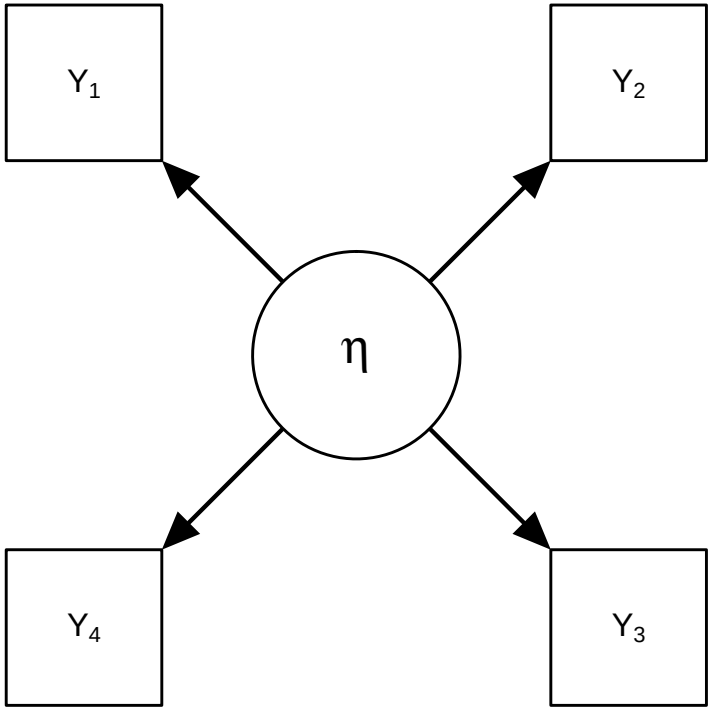


Markov Random Field

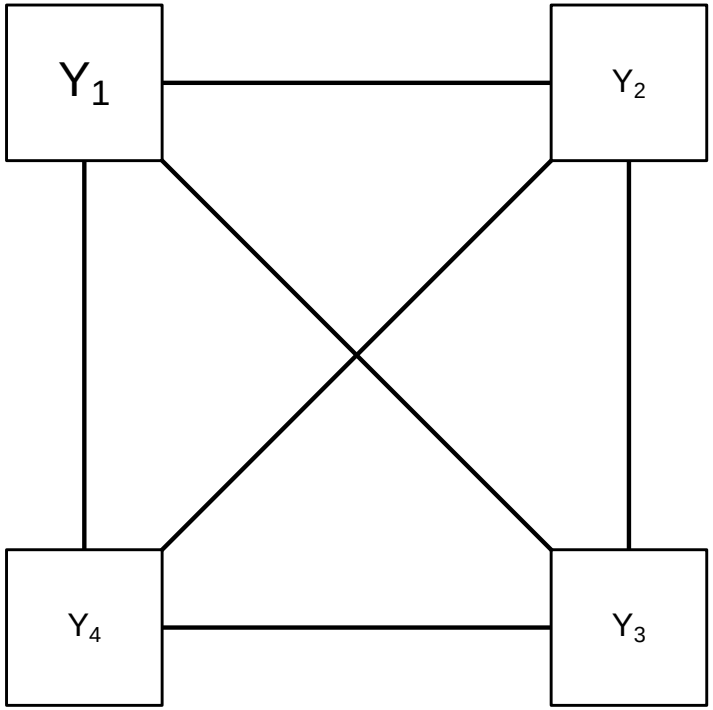


- If we could condition on η

Data generating structure



Markov Random Field



- If we could *not* condition on η
- **Equivalent models**
  - Data generated as a cluster of interacting components can fit a factor model perfectly!

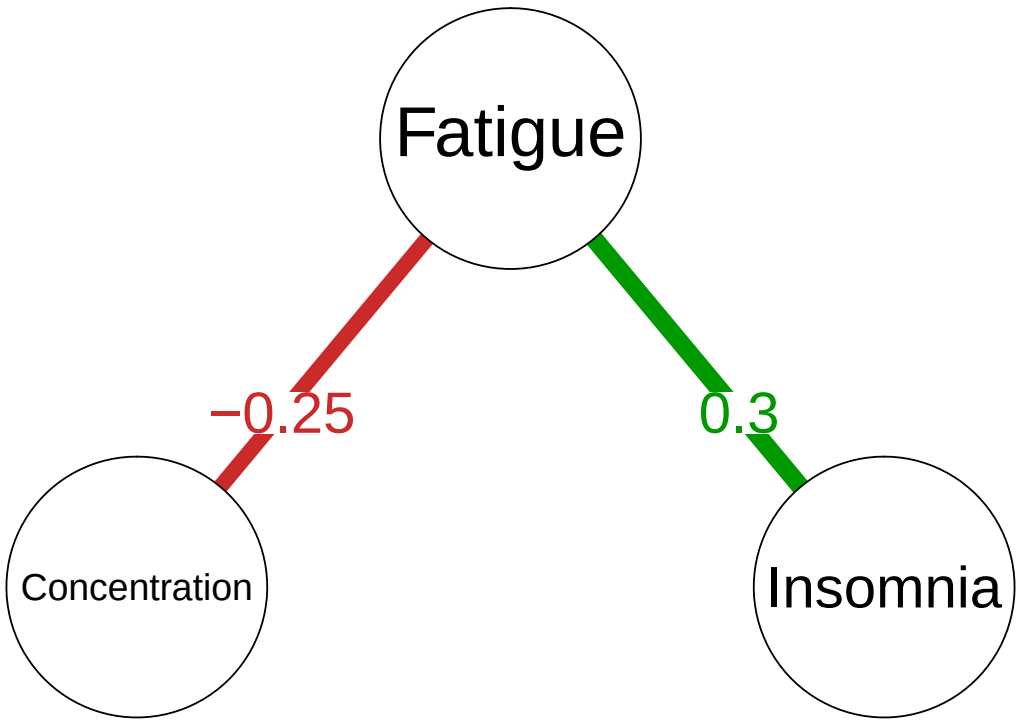
Cross-sectional Data

Cross-sectional Data

- These slides assume multivariate normality
- Every person measured only *once*
- Cases can reasonably be assumed to be *independent*
  - Given IQ has a mean of 100 and SD of 15, does knowing that Peter has an IQ of 90 help us predict better that Sarah had an IQ of 110?
- Because of this assumption, likelihood reduces to a *product*
  - $\boldsymbol{Y} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
  - $f(\boldsymbol{y} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \prod_{p=1}^{N} f\left(\boldsymbol{y}^{(p)} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$

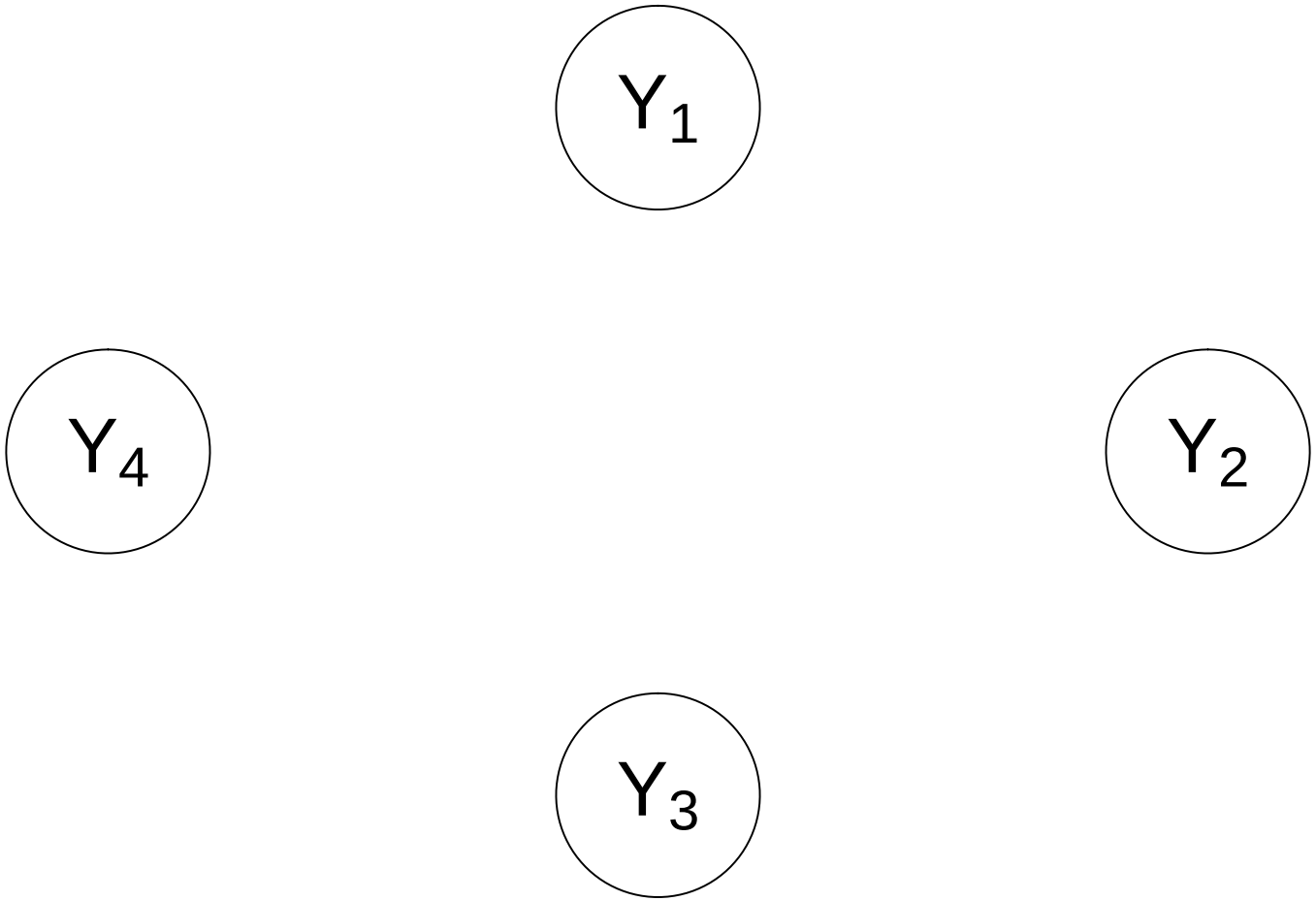
The Gaussian Graphical Model

- $\boldsymbol{\Sigma}$, the variance-covariance matrix, encodes *all* information how variables relate to one-another
- Because of the Schur complement, it also encodes *all* conditional relationships
- We will focus on its *inverse*, $\boldsymbol{K}$:
  - $\boldsymbol{K} = \boldsymbol{\Sigma}^{-1}$
- The inverse variance-covariance matrix is called a Gaussian graphical model (GGM)
  - Encodes an undirected network

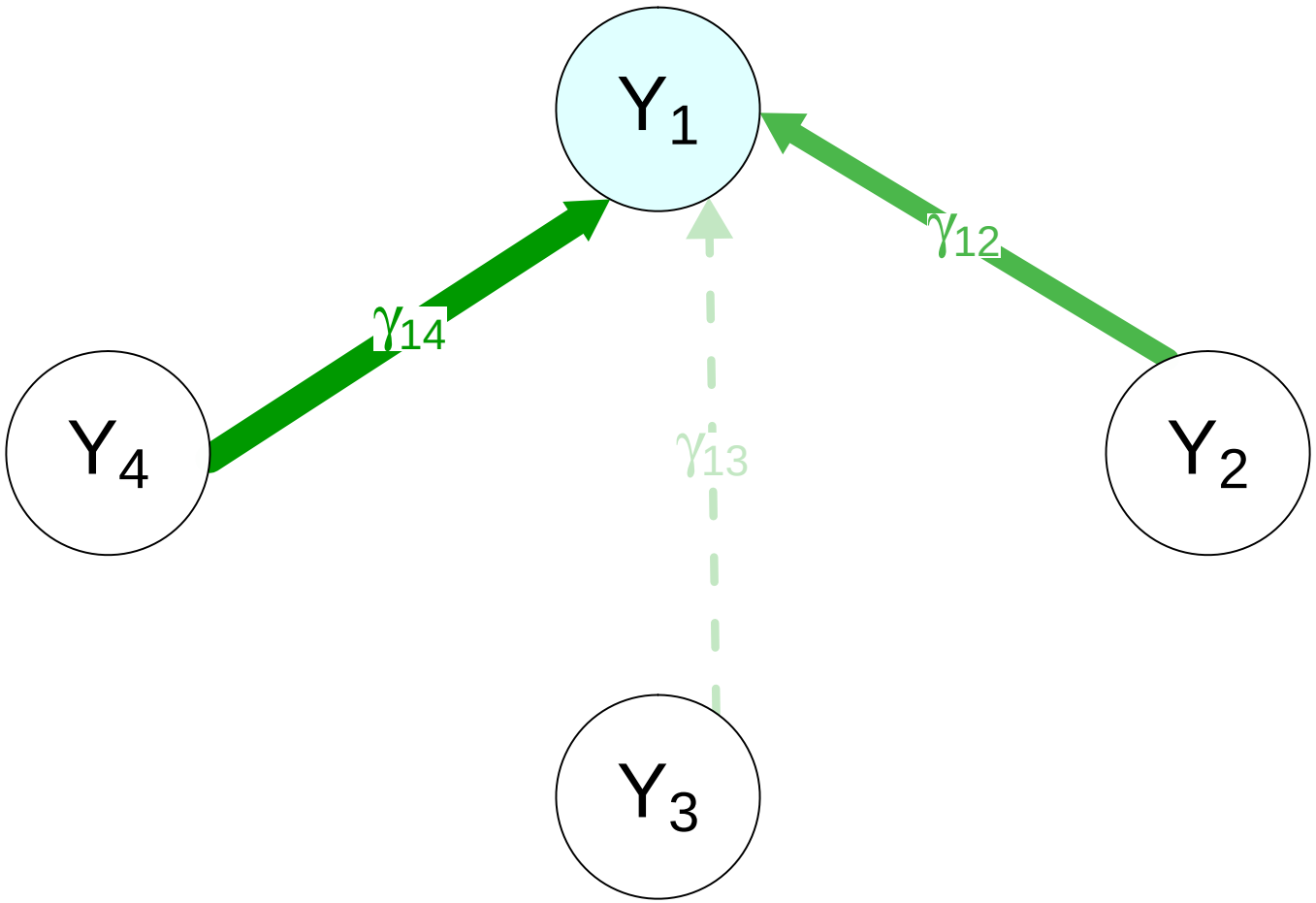


- GGM is a network of *partial correlation coefficients*:
  - $\mathrm{Cor}\left(Y_i, Y_j \mid \boldsymbol{Y}^{-(i,j)}\right) = -\frac{\kappa_{ij}}{\sqrt{\kappa_{ii}}\sqrt{\kappa_{jj}}}$

GGM and Multiple Regressions

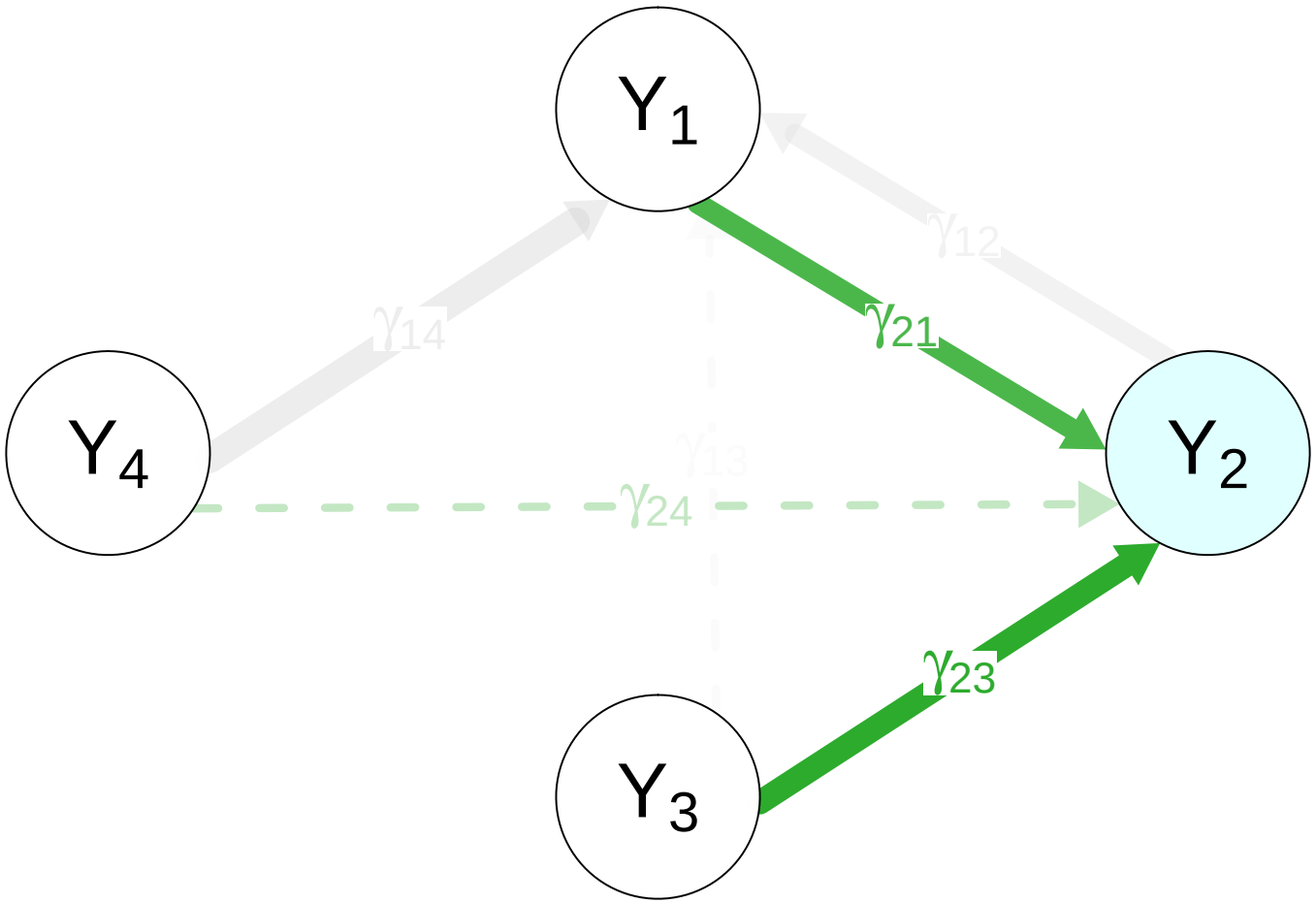


GGM and Multiple Regressions



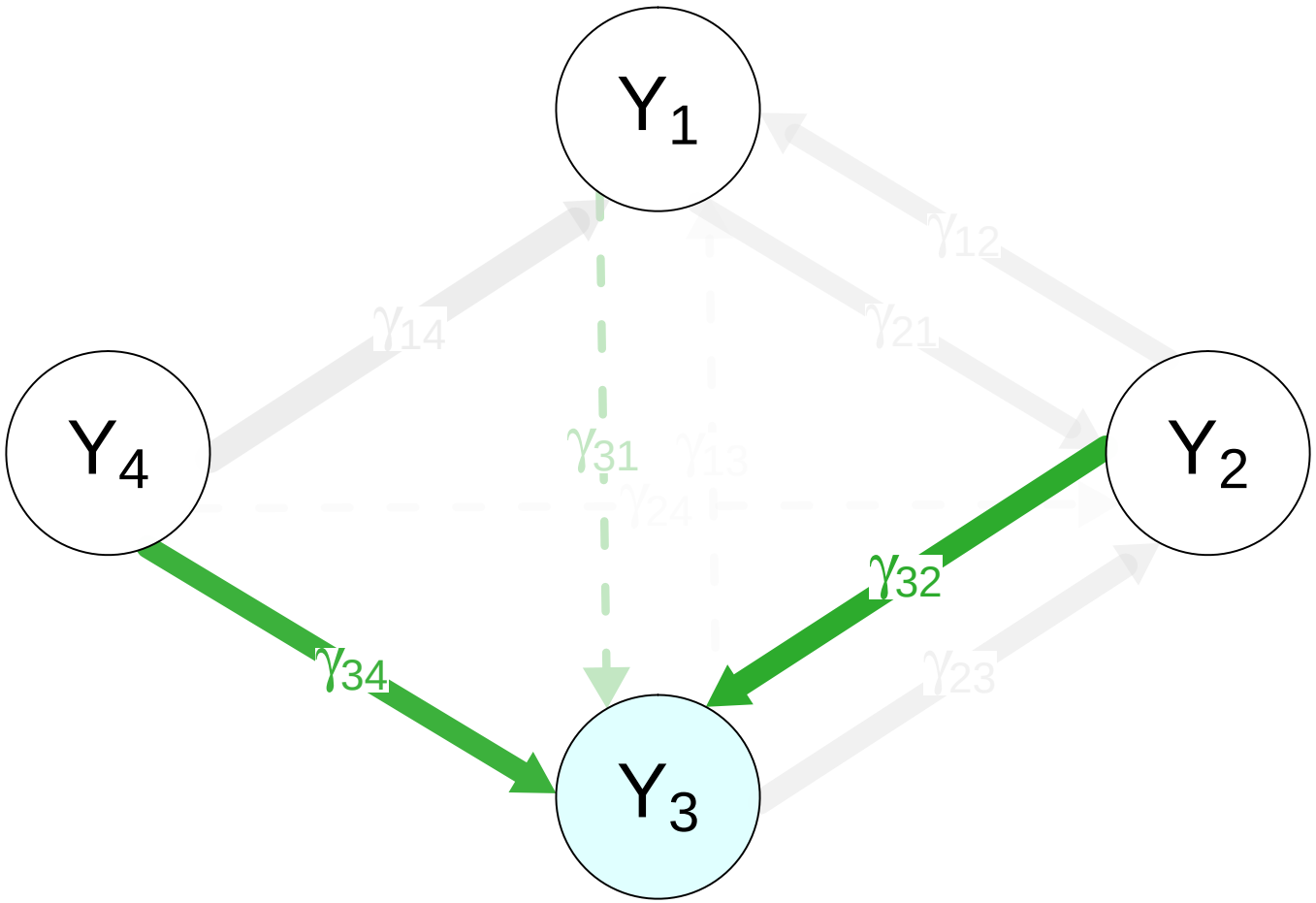
$$y_1 = \tau_1 + \gamma_{12} y_2 + \gamma_{13} y_3 + \gamma_{14} y_4 + \varepsilon_1$$

GGM and Multiple Regressions



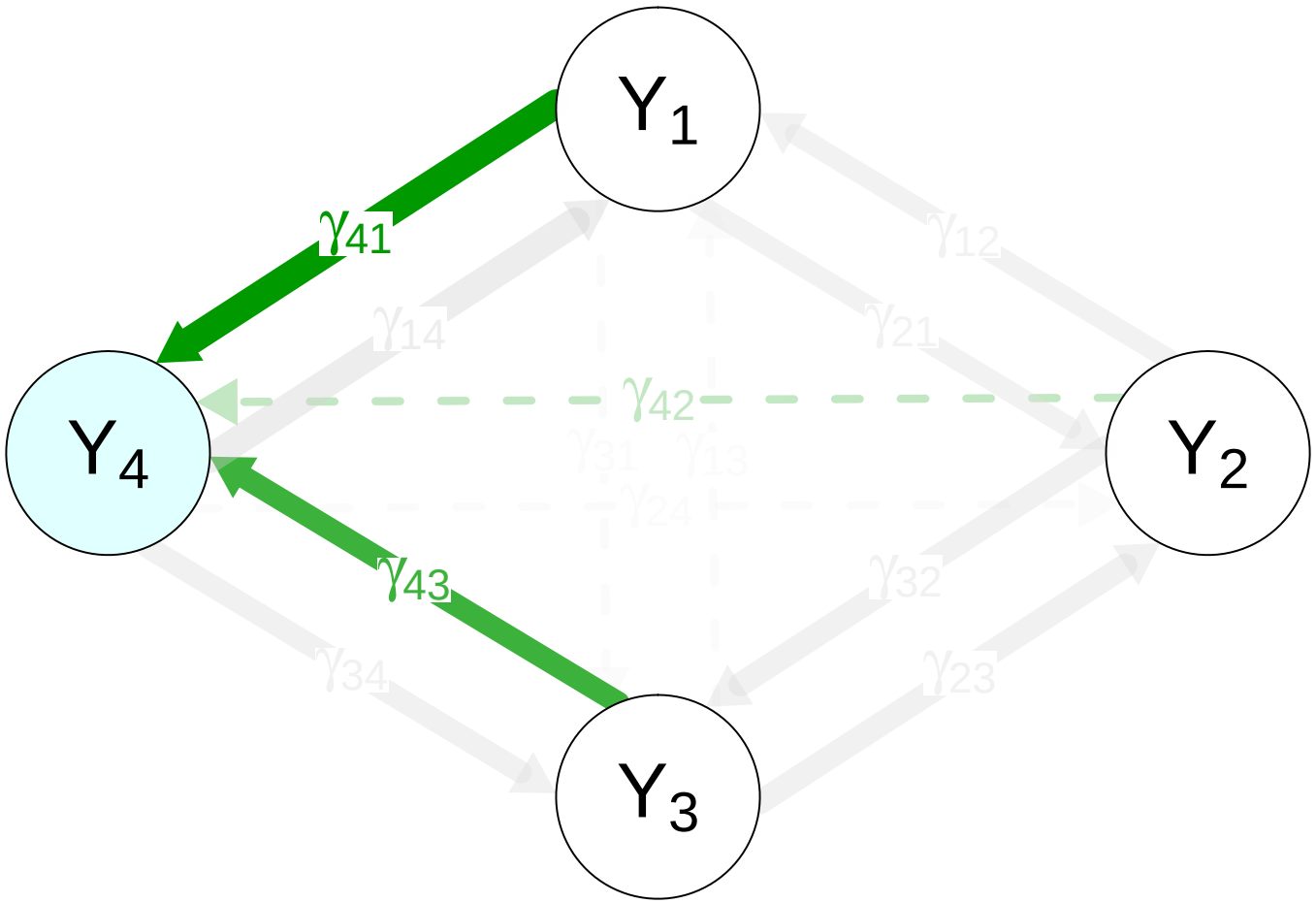
$$y_2 = \tau_2 + \gamma_{21} y_1 + \gamma_{23} y_3 + \gamma_{24} y_4 + \varepsilon_2$$

GGM and Multiple Regressions



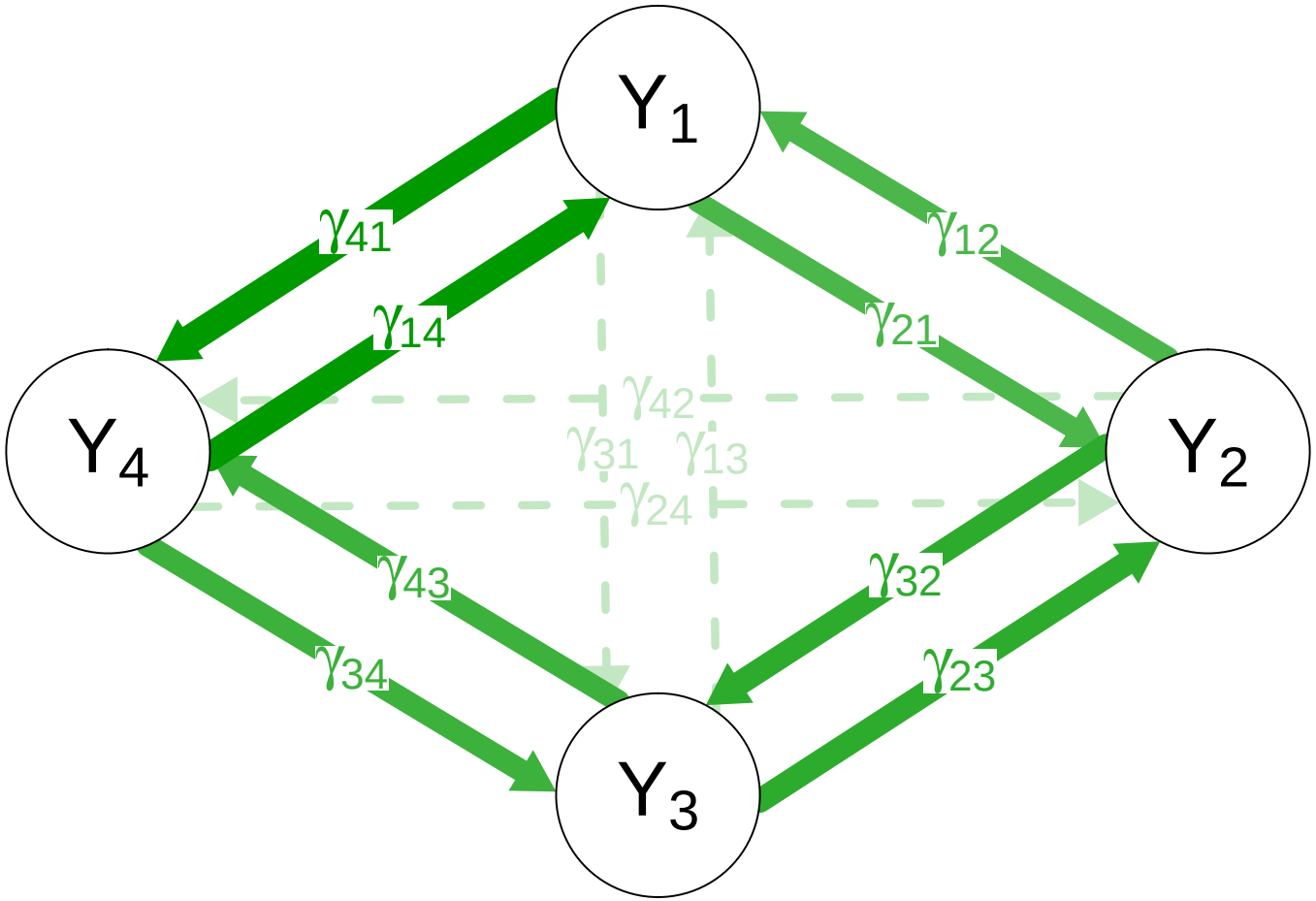
$$y_3 = \tau_3 + \gamma_{31} y_1 + \gamma_{32} y_2 + \gamma_{34} y_4 + \varepsilon_3$$

GGM and Multiple Regressions

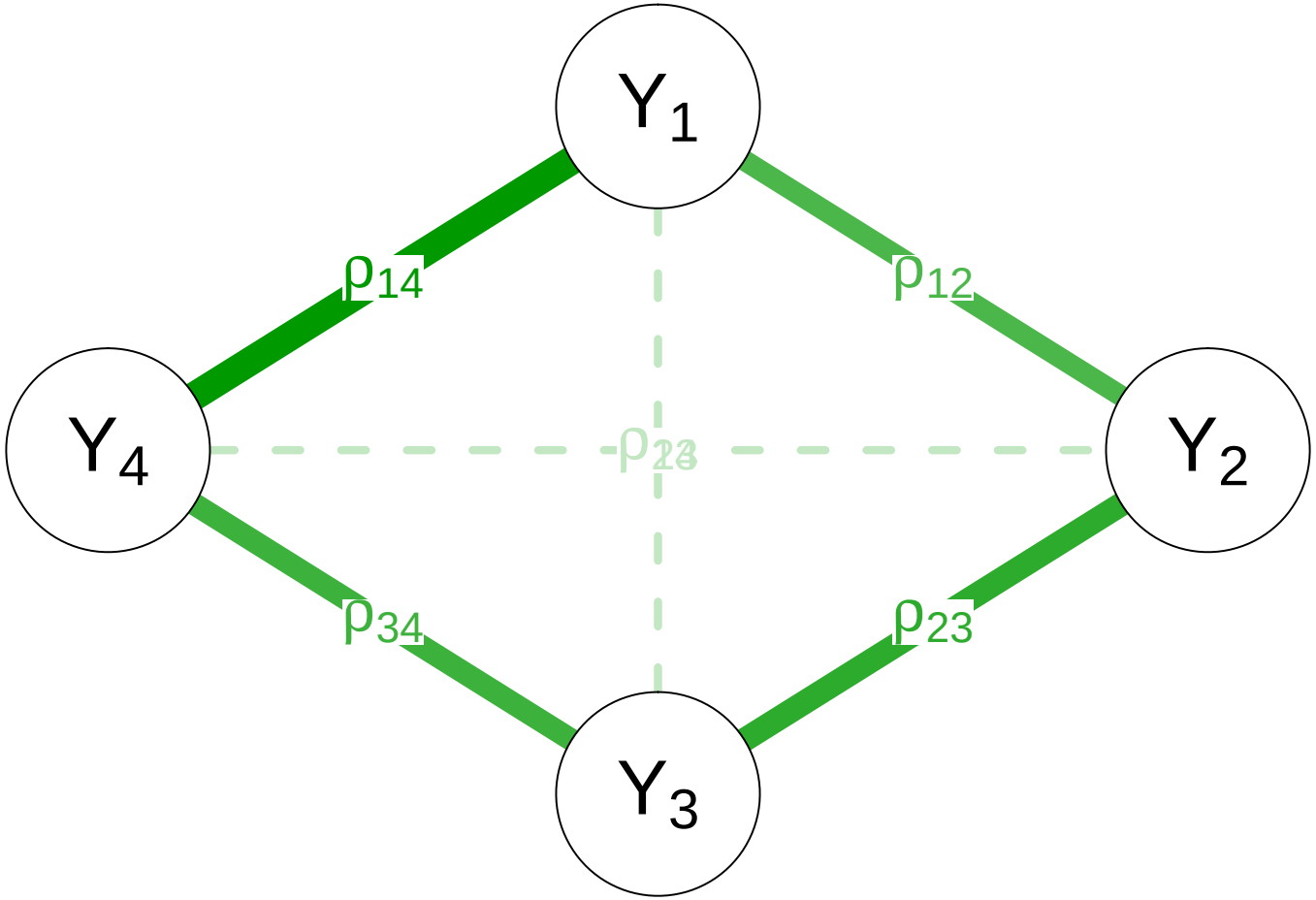


$$y_4 = \tau_4 + \gamma_{41} y_1 + \gamma_{42} y_2 + \gamma_{43} y_3 + \varepsilon_4$$

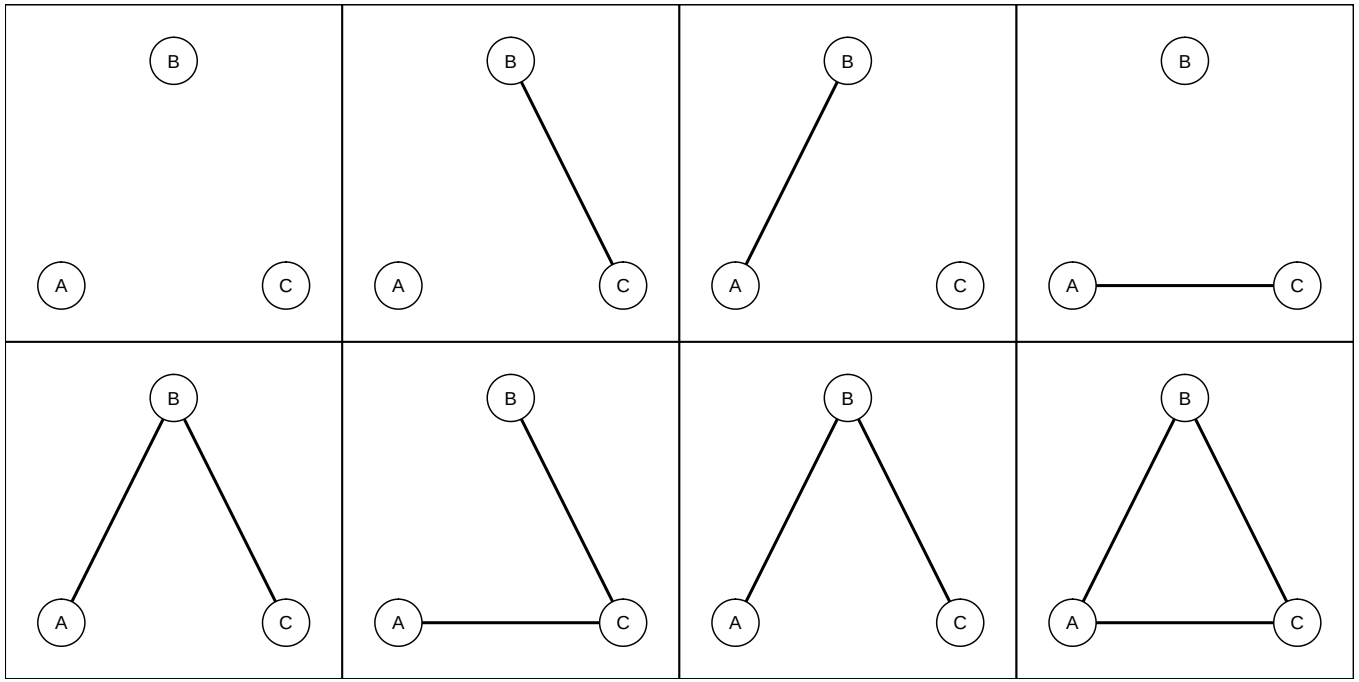
GGM and Multiple Regressions



GGM and Multiple Regressions



$$\rho_{ij} = \frac{\gamma_{ij}\mathrm{Var}\left(\varepsilon_j\right)}{\mathrm{Var}\left(\varepsilon_i\right)} = \frac{\gamma_{ji}\mathrm{Var}\left(\varepsilon_i\right)}{\mathrm{Var}\left(\varepsilon_j\right)} = -\frac{\kappa_{ij}}{\sqrt{\kappa_{ii}}\sqrt{\kappa_{jj}}}$$



- How to select the best model?

arXiv.org > stat > arXiv:1607.01367

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Statistics > Applications

A Tutorial on Regularized Partial Correlation Networks

Sacha Epskamp, Eiko I. Fried

(Submitted on 5 Jul 2016 (v1), last revised 3 Oct 2016 (this version, v4))

Recent years have seen an emergence of network modeling applied to moods, attitudes, and problems in the realm of psychology. In this framework, psychological variables are understood to directly interact with each other rather than being caused by an unobserved latent entity. In this tutorial, we introduce the reader to estimating the most popularly used network model for psychological data: the partial correlation network. We describe how regularization techniques can be used to efficiently estimate a parsimonious and interpretable network structure on cross-sectional data. We show how to perform these analyses in R and demonstrate the method in an empirical example on post-traumatic stress disorder data. In addition, we discuss the effect of the hyperparameter that needs to be manually set by the researcher and provide a checklist with potential solutions for problems often arise when estimating regularized partial correlation networks.

Comments: Submitted for publication to journal Psychological Methods
Subjects: **Applications (stat.AP)**; Methodology (stat.ME)
Cite as: arXiv:1607.01367 [stat.AP]
(or arXiv:1607.01367v4 [stat.AP] for this version)

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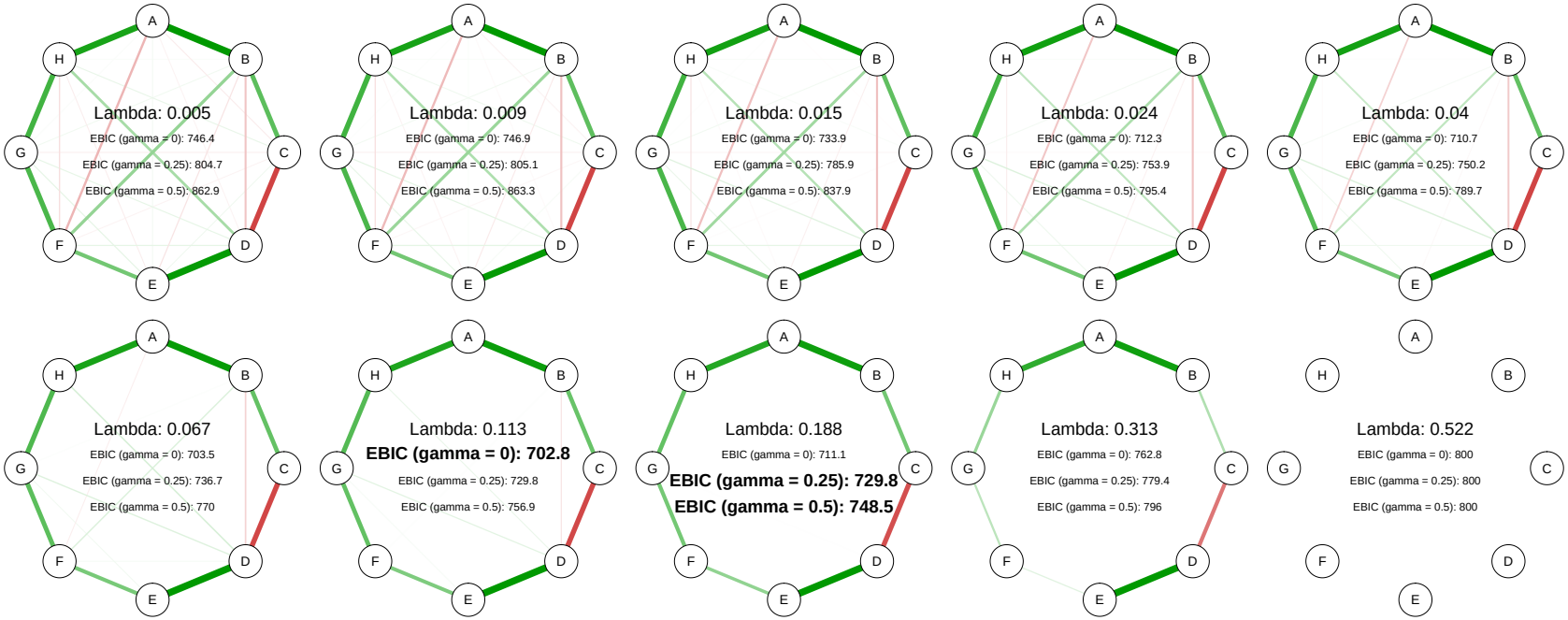
Lambda: 0.005
EBIC (gamma = 0): 746.4
EBIC (gamma = 0.25): 804.7
EBIC (gamma = 0.5): 862.9
Lambda: 0.009
EBIC (gamma = 0): 746.9
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Lambda: 0.015
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Lambda: 0.522
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EBIC (gamma = 0.25): 800
EBIC (gamma = 0.5): 800
A
B
C
D
E
F
G
H

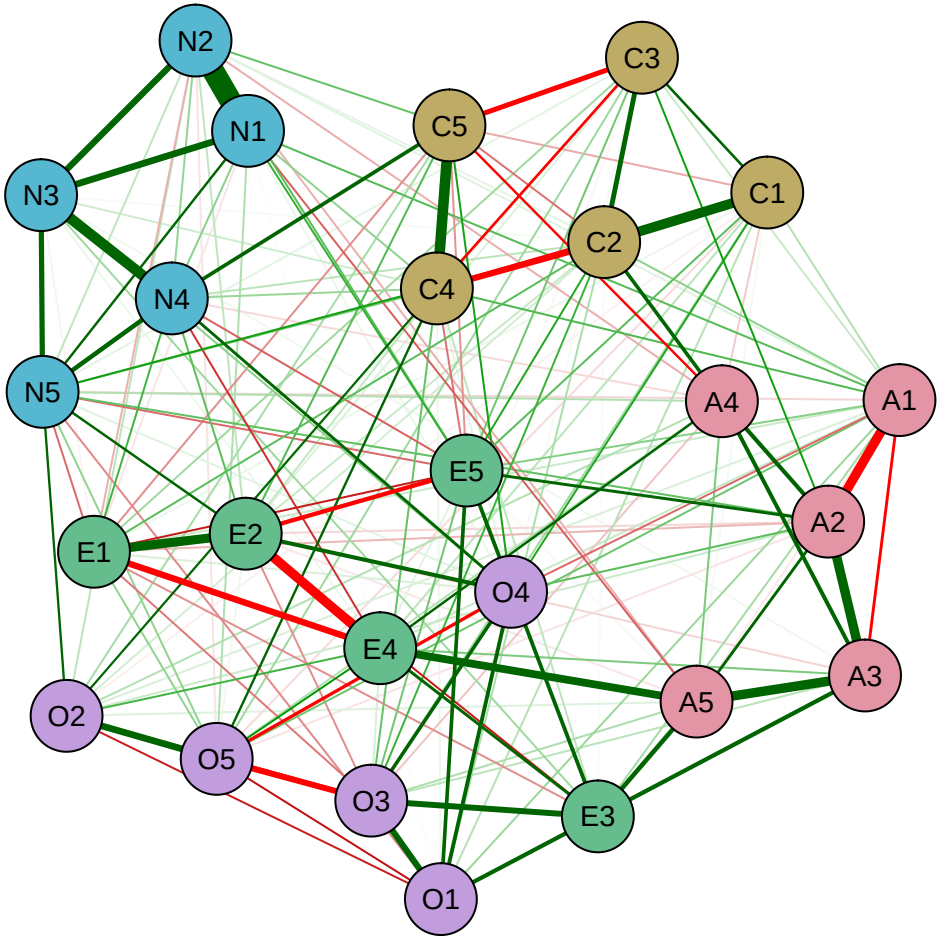
Estimation

- GGM can be computed using *qgraph*
- Ordinal data
  - Use polychoric correlations (`cor_auto` in *qgraph*) as input
- Non-normal continuous data
  - Transform variables first (`huge.npn` in *huge*)
- Binary data
  - Use *IsingFit*
- Mixed variables
  - Use *mgm*

Emperical Example: Personality

BFI dataset from the pych package:

- 25 items
- 2800 subjects
- Five items for each of the five central personality traits



Agreeableness
- A1: Am indifferent to the feelings of others.
- A2: Inquire about others' well−being.
- A3: Know how to comfort others.
- A4: Love children.
- A5: Make people feel at ease.

Conscientiousness
- C1: Am exacting in my work.
- C2: Continue until everything is perfect.
- C3: Do things according to a plan.
- C4: Do things in a half−way manner.
- C5: Waste my time.

Extraversion
- E1: Don't talk a lot.
- E2: Find it difficult to approach others.
- E3: Know how to captivate people.
- E4: Make friends easily.
- E5: Take charge.

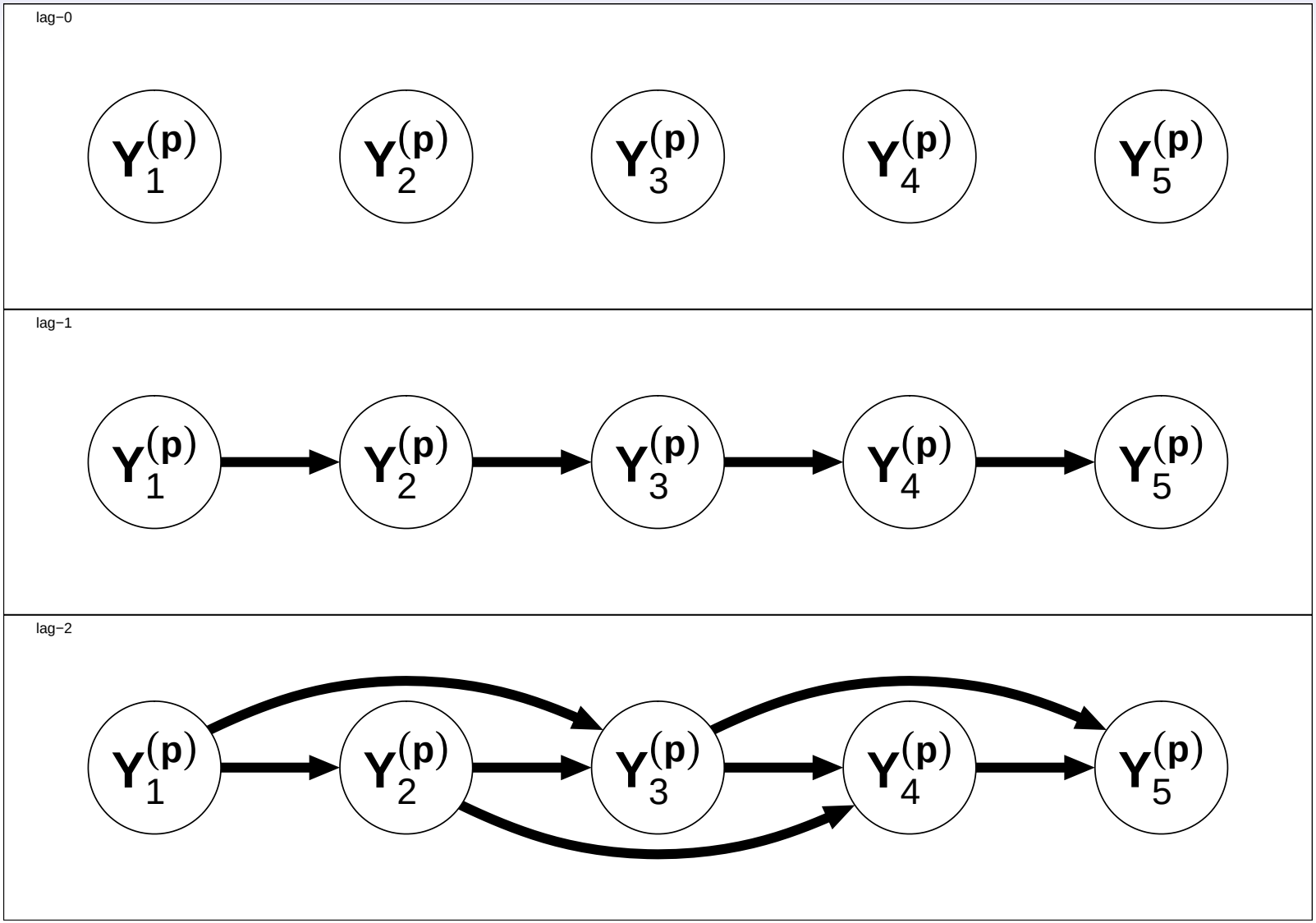
Neuroticism
- N1: Get angry easily.
- N2: Get irritated easily.
- N3: Have frequent mood swings.
- N4: Often feel blue.
- N5: Panic easily.

Openness
- O1: Am full of ideas.
- O2: Avoid difficult reading material.
- O3: Carry the conversation to a higher level.
- O4: Spend time reflecting on things.
- O5: Will not probe deeply into a subject.

Time-series: $n = 1$

Time-series Data

- One person measured several times in a short period
- Cases can **not** reasonably be assumed to be *independent*
  - Knowing someone's level of fatigue at a time point helps predict his or her level of fatigue at the next time point.
- Likelihood not easy to compute without three assumptions:
  - The time-series factorize according to a graph
  - The model does not change over time
  - The first measurement is exogenous

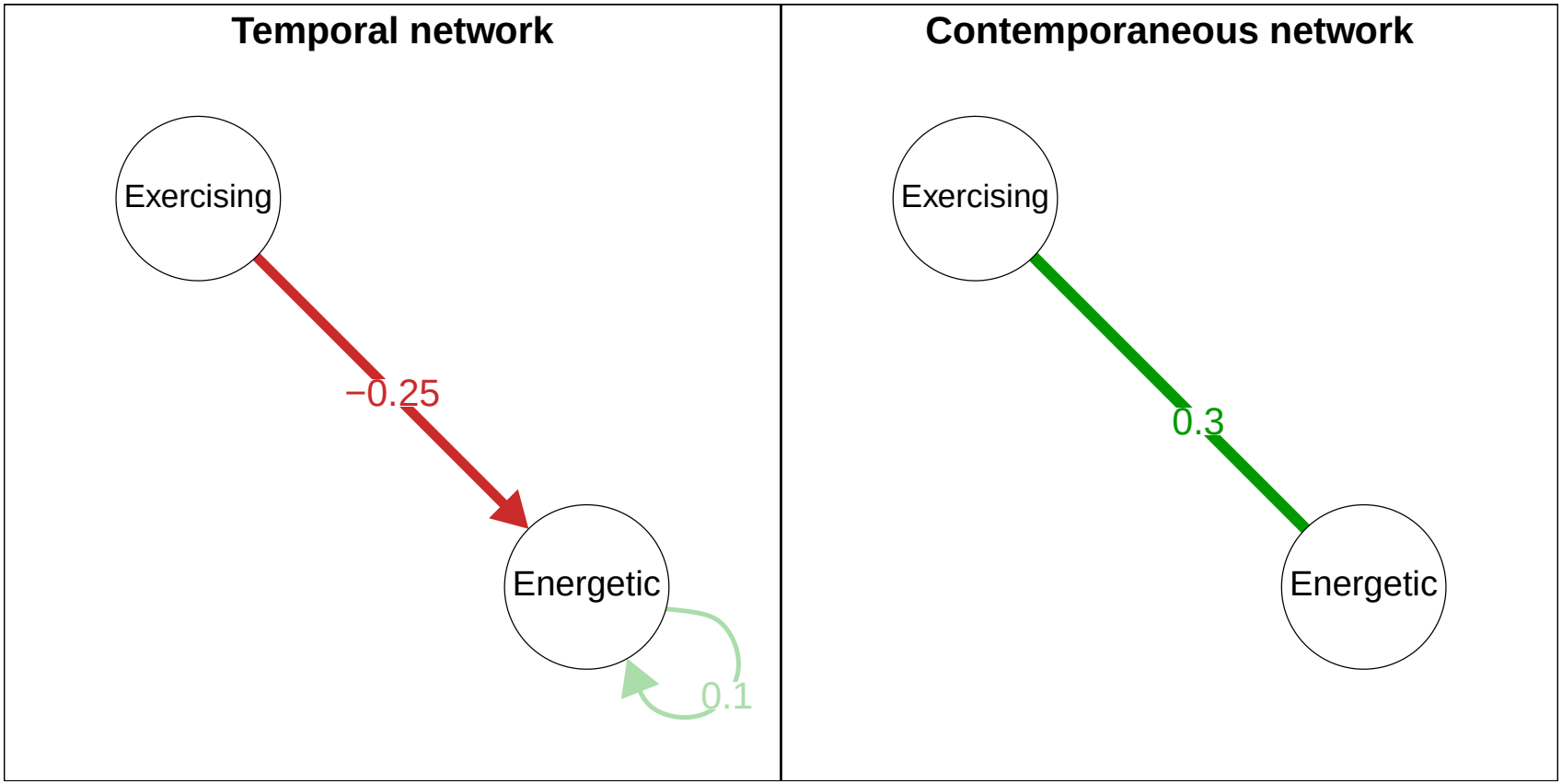


- We will use the lag-1 factorization

Vector Auto-regression

$$\boldsymbol{Y}_t \mid \boldsymbol{y}_{t-1} \sim N\left(\boldsymbol{\mu} + \boldsymbol{B}\left(\boldsymbol{y}_{t-1} - \boldsymbol{\mu}\right), \boldsymbol{\Theta}\right)$$

- $\boldsymbol{B}$ encodes the *temporal network*
  - Granger causality
- $\boldsymbol{\Theta}^{-1}$ encodes the *contemporaneous network*
  - GGM
- The sample means can be used as plugin to center the predictors


Temporal network
Exercising
−0.25
Energetic
0.1
Contemporaneous network
Exercising
0.3
Energetic

Contemporaneous Causation

- Many causal effects likely faster than the time-window of measurement
  - Somatic arousal $\rightarrow$ anticipation of panic attack $\rightarrow$ anxiety
- These can be caught in a contemporaneous network of **partial correlations**
- Thus, the contemporaneous network can also be seen to highlight potential causal relationships
- As the contemporaneous network is the GGM, the temporal network can be seen as a correction for dependent measurements in estimating the GGM

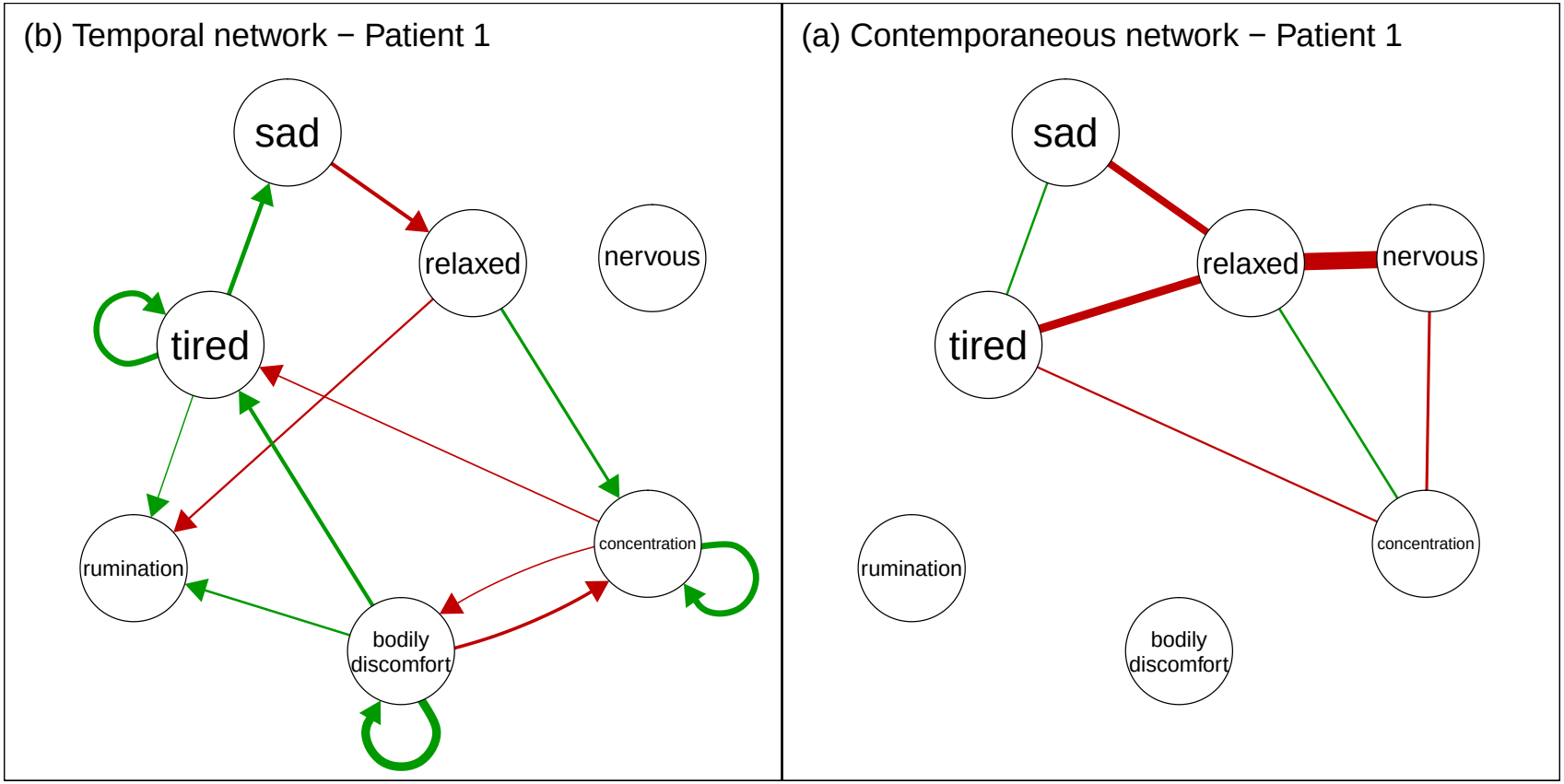
- Estimation straightforward using multiple regression
- For model selection, we use the graphical VAR model
  - Wild et al. (2010). A graphical vector autoregressive modelling approach to the analysis of electronic diary data. *BMC Medical Research Methodology 10 (1):* 28.
- Estimation via LASSO regularization, using EBIC to select optimal tuning parameter
  - Abegaz & Wit (2013). Sparse Time Series Chain Graphical Models for Reconstructing Genetic Networks. *Biostatistics*: kxt005.
  - Rothman, Elizaveta, & Zhu (2010). Sparse Multivariate Regression with Covariance Estimation. *Journal of Computational and Graphical Statistics 19 (4)*: 947–62.
- We implemented these methods in the R package `graphicalVAR`
- Also implemented in `sparseTSCGM`

Personalized Network Modeling in Psychopathology: The Importance of Contemporaneous and Temporal Connections

Sacha Epskamp[1], Claudia D. van Borkulo[1], Date C. van der Veen[2], Michelle N. Servaas[2], Adela-Maria Isvoranu[1], Harriëtte Riese[2], Angelique O.J. Cramer[1]

1. University of Amsterdam, Department of Psychological Methods
2. University of Groningen, University Medical Center Groningen, Department of Psychiatry, Interdisciplinary Center for Psychopathology and Emotion Regulation

Personalized Networks in Clinical Practice



- Contemporaneous network: conditional concentration given $t - 1$
- Temporal network: regression coefficients between $t - 1$ and t

Time-series: $n > 1$

Multi-level VAR

- Each subject is assumed to have their own temporal and contemporaneous VAR model
- VAR parameters come from distribution
  - Fixed effect
  - Random effect
- Bringmann, L. F., Vissers, N., Wichers, M., Geschwind, N., Kuppens, P., Peeters, F., ... & Tuerlinckx, F. (2013). A network approach to psychopathology: new insights into clinical longitudinal data. *PloS one, 8(4)*, e60188.

Multi-level VAR

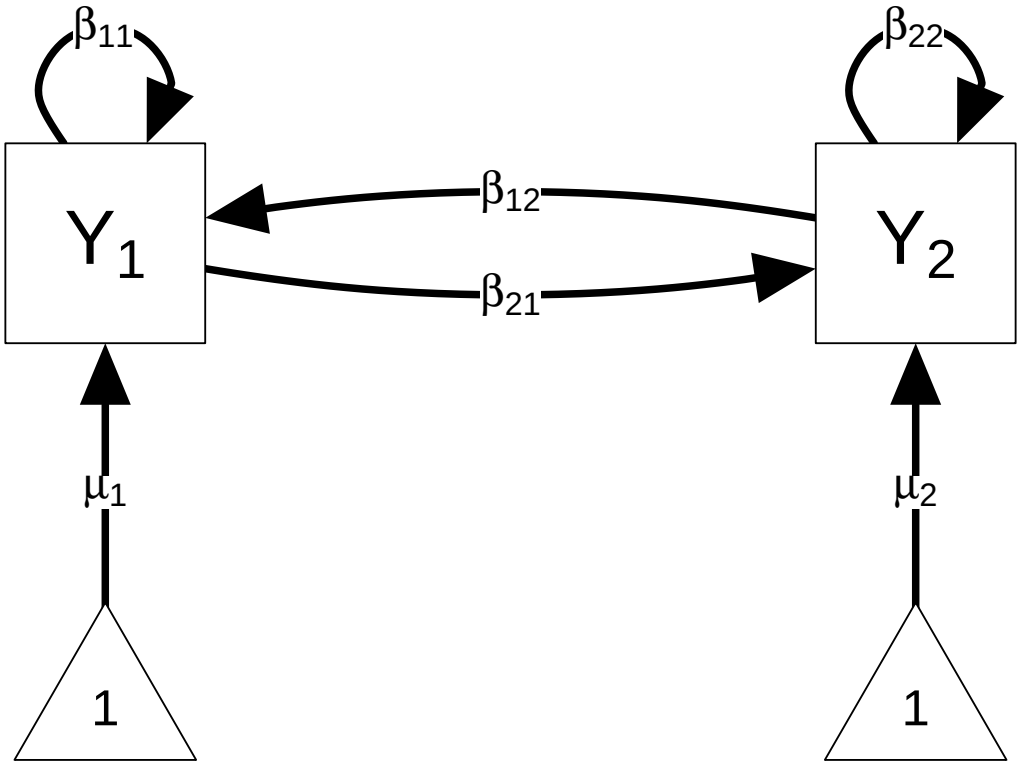
Adding superscript p for subject. Level 1 model:

$$\boldsymbol{Y}_t^{(p)} \mid \boldsymbol{y}_t^{(p)} = N\left(\boldsymbol{\mu}^{(p)} + \boldsymbol{B}^{(p)}\boldsymbol{y}_{t-1}^{(p)}, \boldsymbol{\Theta}^{(p)}\right)$$

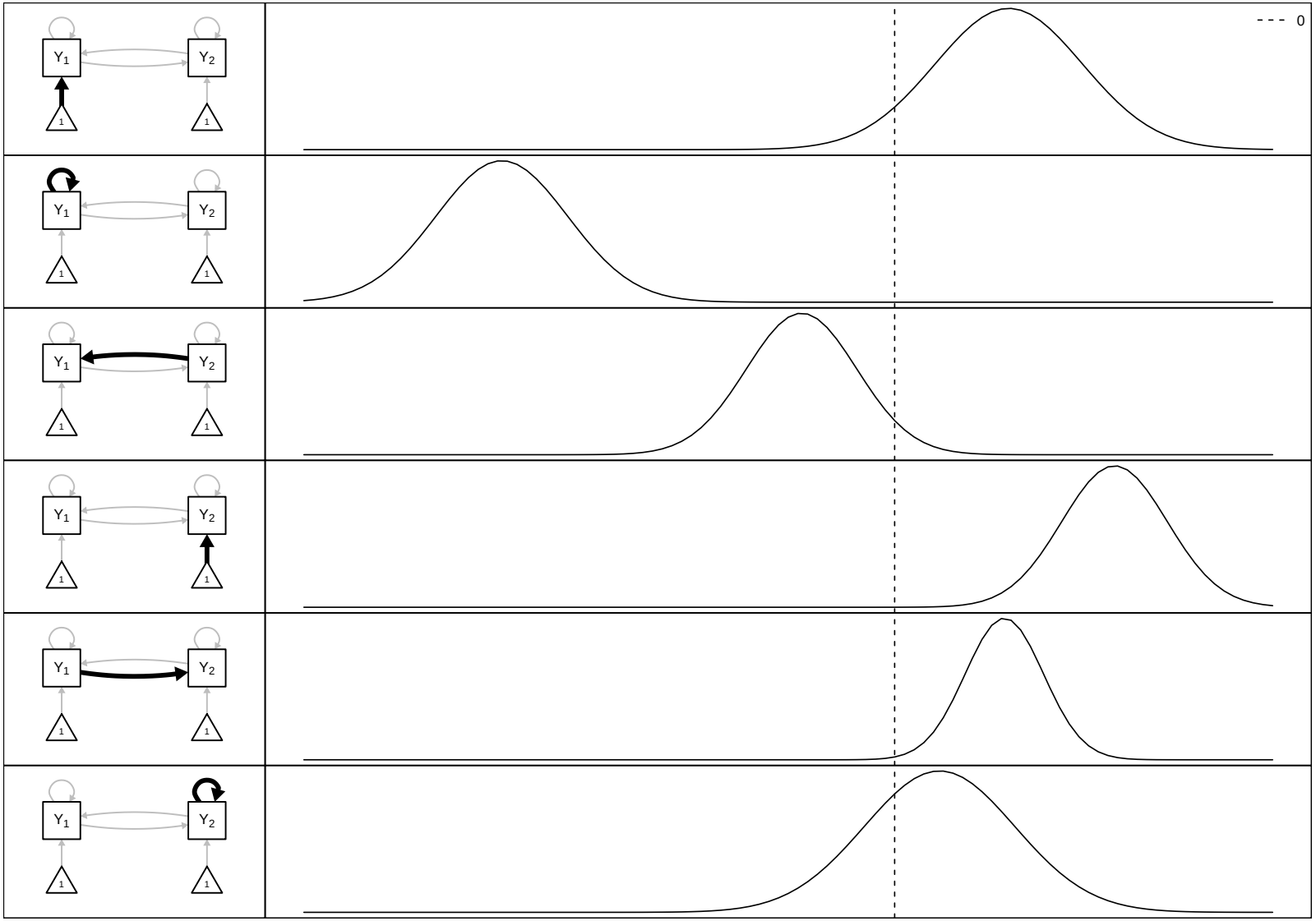
Level 2 model:

$$\begin{bmatrix} \boldsymbol{\mu}^{(p)} \\ \mathrm{Vec}\left(\boldsymbol{B}^{(p)}\right) \end{bmatrix} \sim N\left(\boldsymbol{f}, \boldsymbol{\Omega}\right).$$

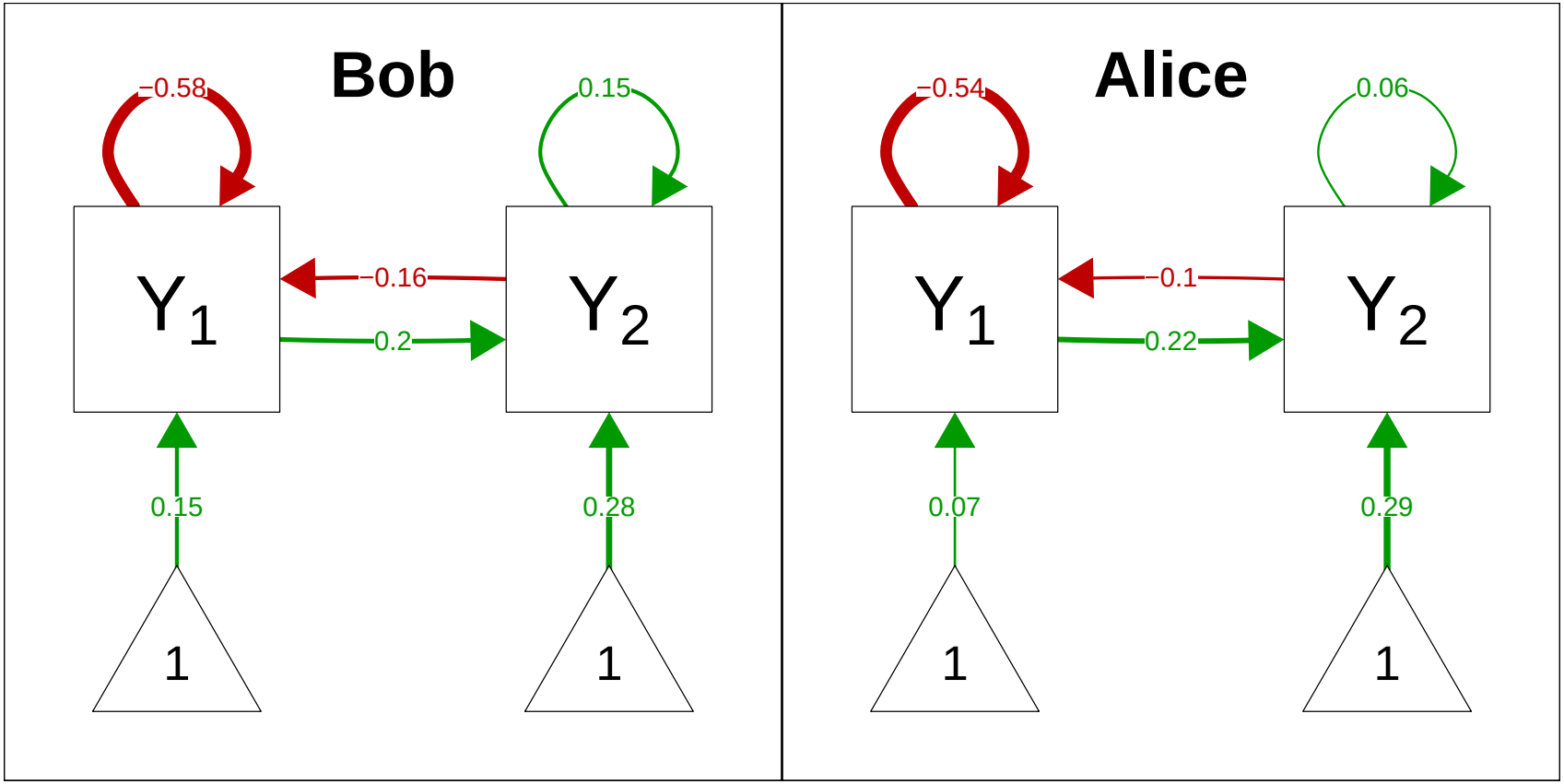
$\boldsymbol{f}$ encodes fixed effects and $\boldsymbol{\Omega}$ the distribution of random effects.


β11
β22
Y1
Y2
β12
β21
μ1
μ2
1
1

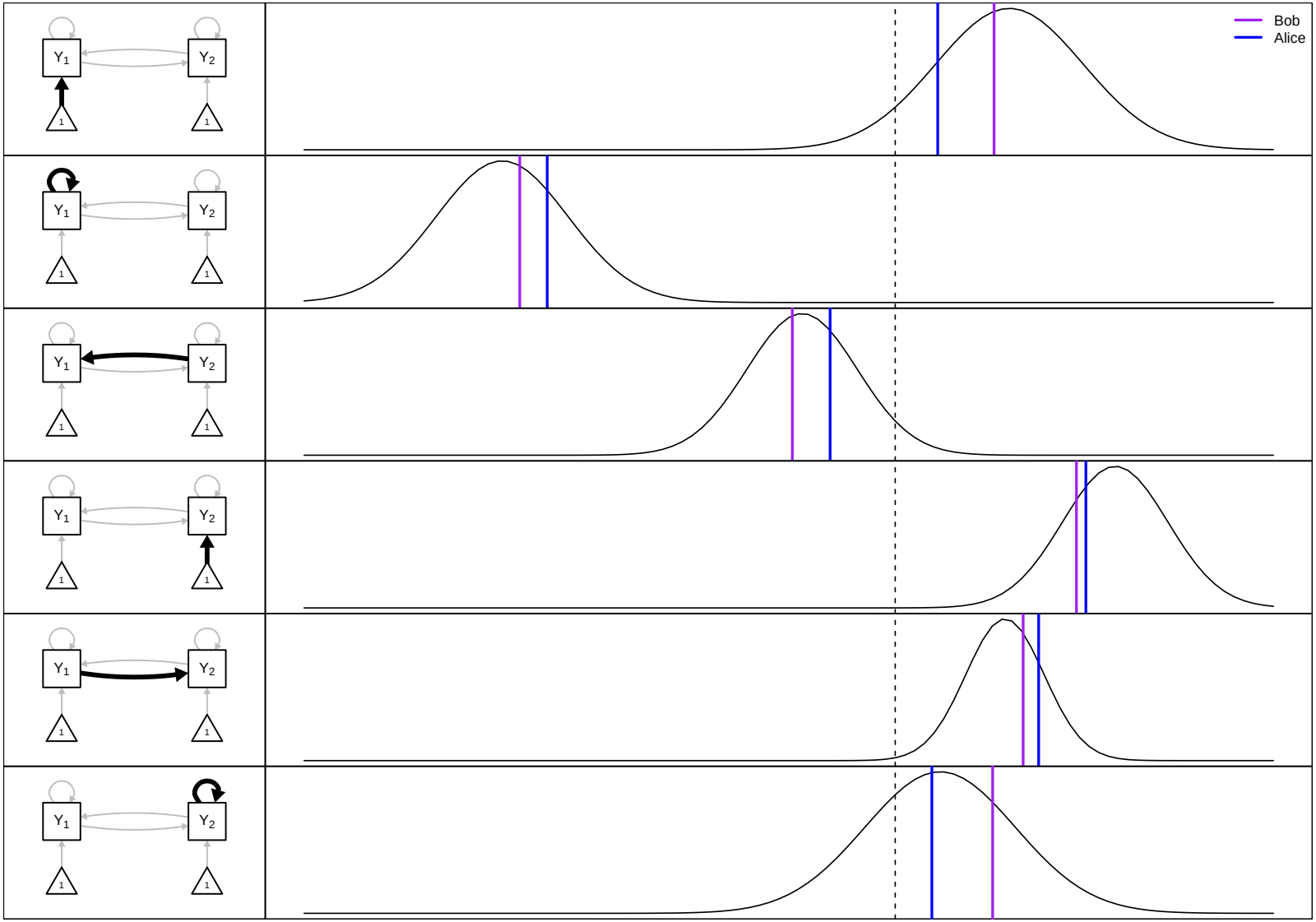
Each Parameter has a Distribution



Individual Networks



Random Effects



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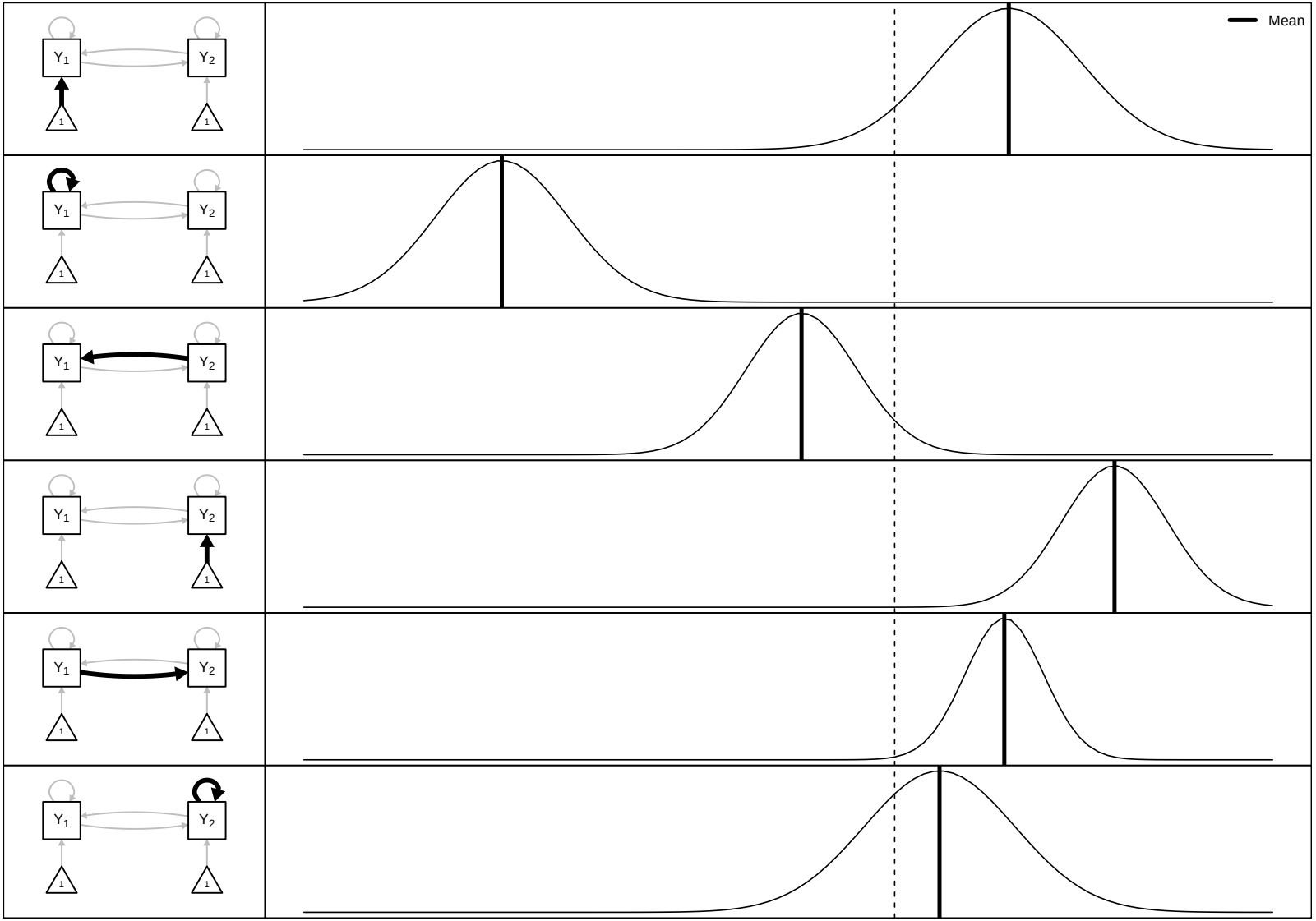

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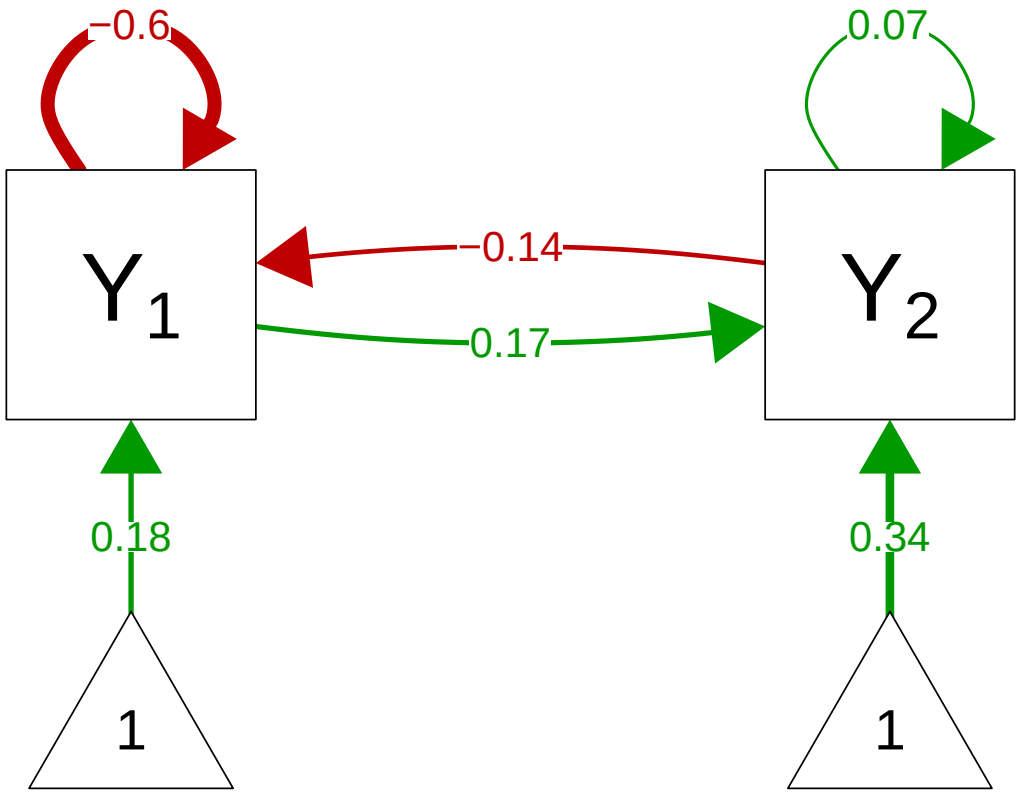
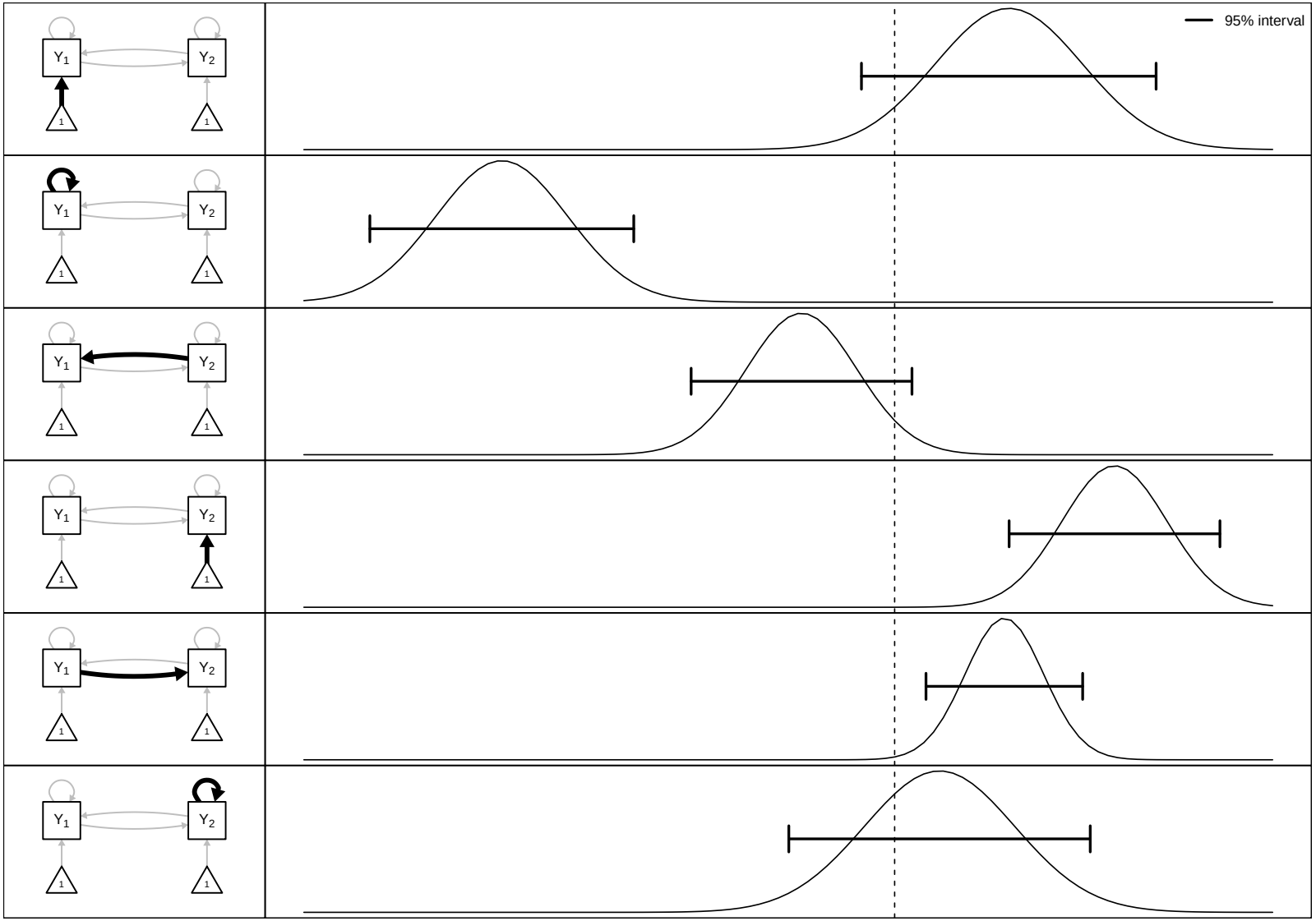
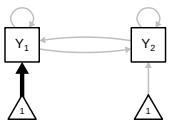
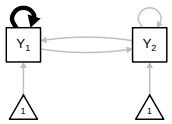
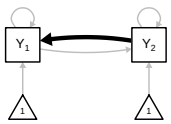
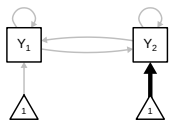
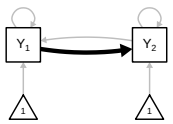
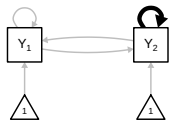
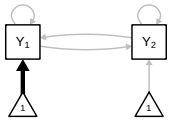
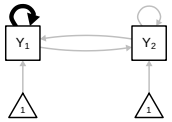
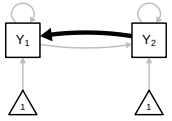
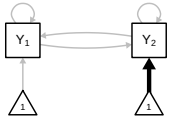
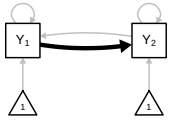
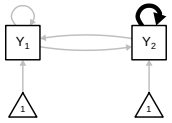


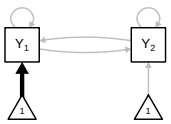
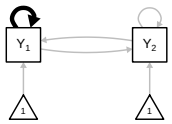
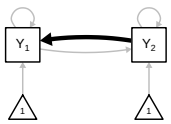
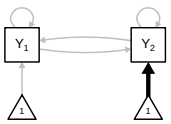
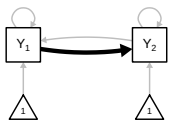
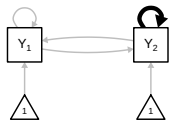
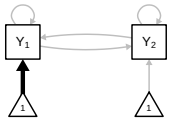
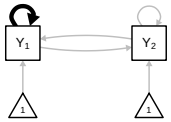
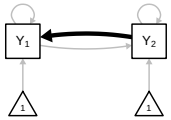
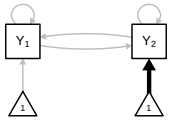
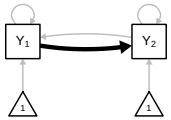
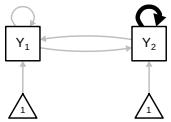
Individual Differences



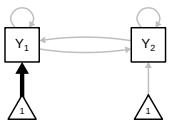
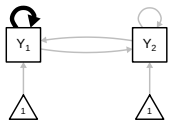
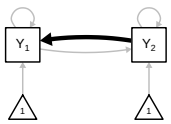
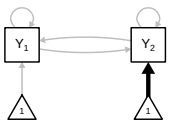
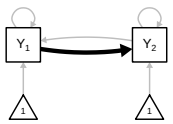
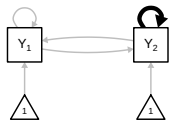
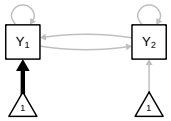
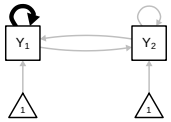
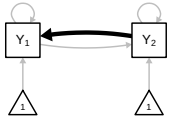
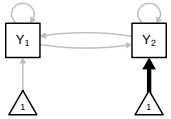
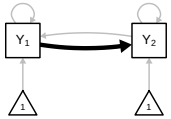
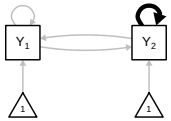
Parameter correlation Matrix

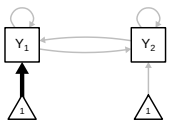
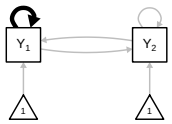
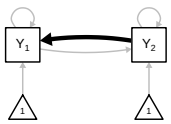
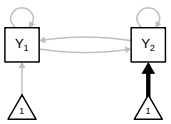
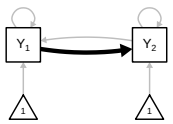
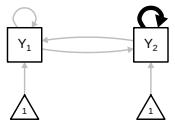
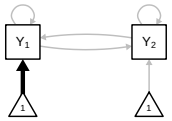
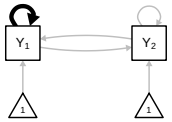
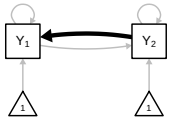
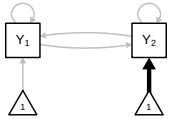
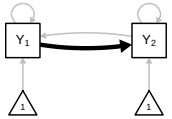
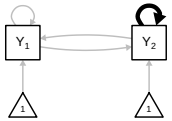
Parameter Correlation Matrix

						
	1	0.01	−0.54	0.21	−0.18	0.46
	0.01	1	−0.24	−0.66	0.77	0.06
	−0.54	−0.24	1	−0.12	0.34	−0.75
	0.21	−0.66	−0.12	1	−0.76	0.1
	−0.18	0.77	0.34	−0.76	1	−0.18
	0.46	0.06	−0.75	0.1	−0.18	1

Stability

						
	1	0.01	−0.54	0.21	−0.18	0.46
	0.01	1	−0.24	−0.66	0.77	0.06
	−0.54	−0.24	1	−0.12	0.34	−0.75
	0.21	−0.66	−0.12	1	−0.76	0.1
	−0.18	0.77	0.34	−0.76	1	−0.18
	0.46	0.06	−0.75	0.1	−0.18	1

Connectivity

						
	1	0.01	−0.54	0.21	−0.18	0.46
	0.01	1	−0.24	−0.66	0.77	0.06
	−0.54	−0.24	1	−0.12	0.34	−0.75
	0.21	−0.66	−0.12	1	−0.76	0.1
	−0.18	0.77	0.34	−0.76	1	−0.18
	0.46	0.06	−0.75	0.1	−0.18	1

Brief Empirical Report

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Emotion-Network Density in Major Depressive Disorder

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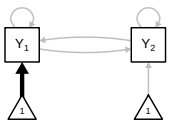
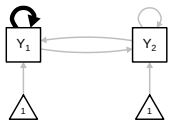
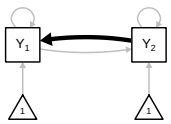
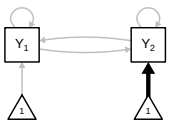
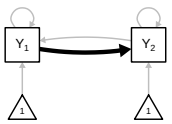
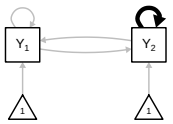
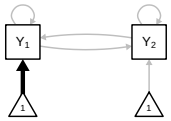
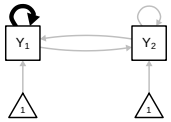
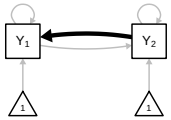
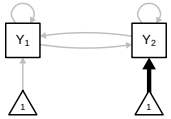
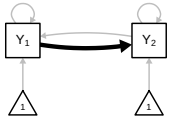
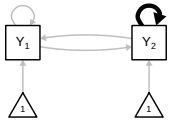
[1]Department of Psychology, KU Leuven; [2]Department of Psychology, Stanford University; [3]Department of Psychology, Washington University in St. Louis; [4]Max Planck Institute for Human Development; [5]School of Education, University of California, Irvine; [6]MIND Research Institute, Irvine, California; and [7]Department of Psychology, University of Michigan, Ann Arbor

Between-subject Effects

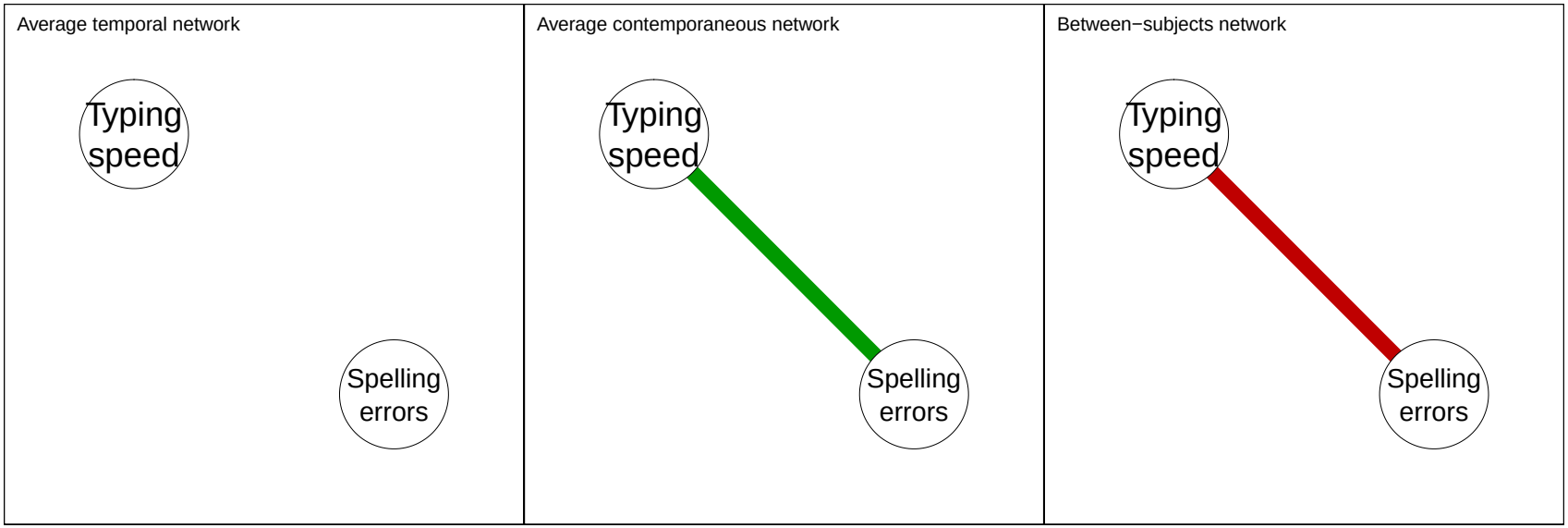
						
	1	0.01	−0.54	0.21	−0.18	0.46
	0.01	1	−0.24	−0.66	0.77	0.06
	−0.54	−0.24	1	−0.12	0.34	−0.75
	0.21	−0.66	−0.12	1	−0.76	0.1
	−0.18	0.77	0.34	−0.76	1	−0.18
	0.46	0.06	−0.75	0.1	−0.18	1

Between-subjects Network

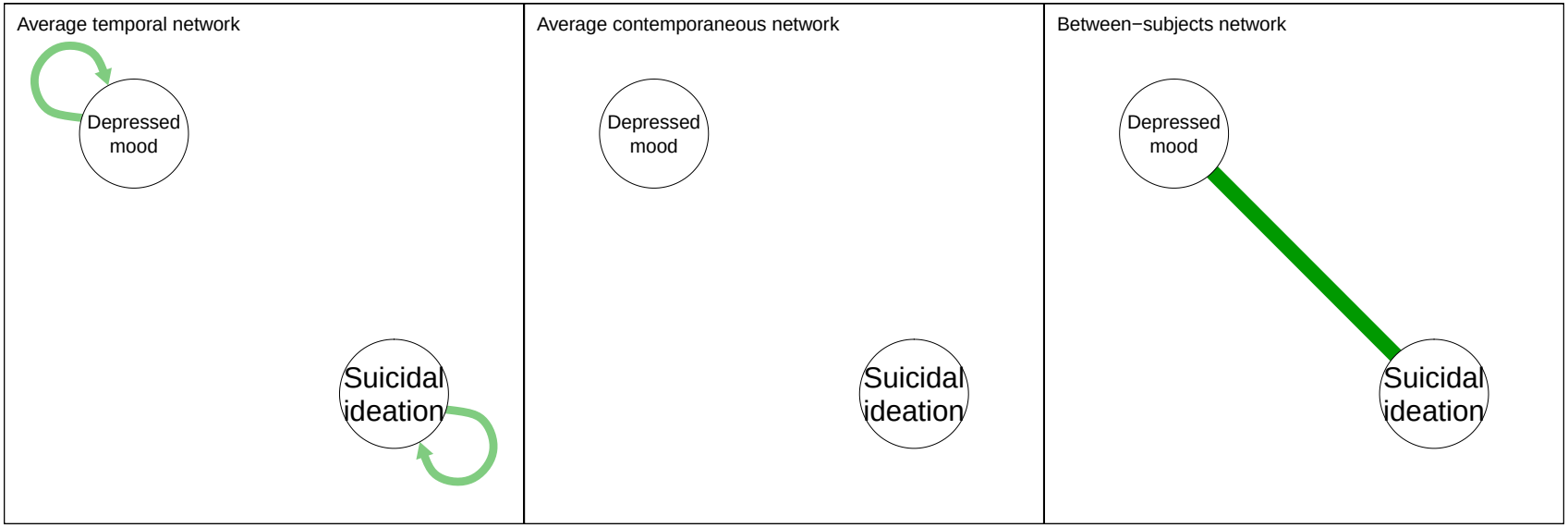
The random effects variance-covariance matrix can be divided in four blocks:

$$\begin{bmatrix} \boldsymbol{R_\mu} \\ \boldsymbol{R_B} \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} \boldsymbol{\Omega_\mu} & \boldsymbol{\Omega_{\mu B}} \\ \boldsymbol{\Omega_{B\mu}} & \boldsymbol{\Omega_B} \end{bmatrix}\right).$$

- Block $\boldsymbol{\Omega_\mu}$ encodes the between-subject relationships between *means*
- These can be used to estimate a GGM
  - Between-subjects network of **partial correlations**



Example based on Hamaker, E. L. (2012). Why Researchers Should Think 'Within-Person': A Paradigmatic Rationale. *Handbook of Research Methods for Studying Daily Life*. The Guilford Press New York, NY, 43–61.



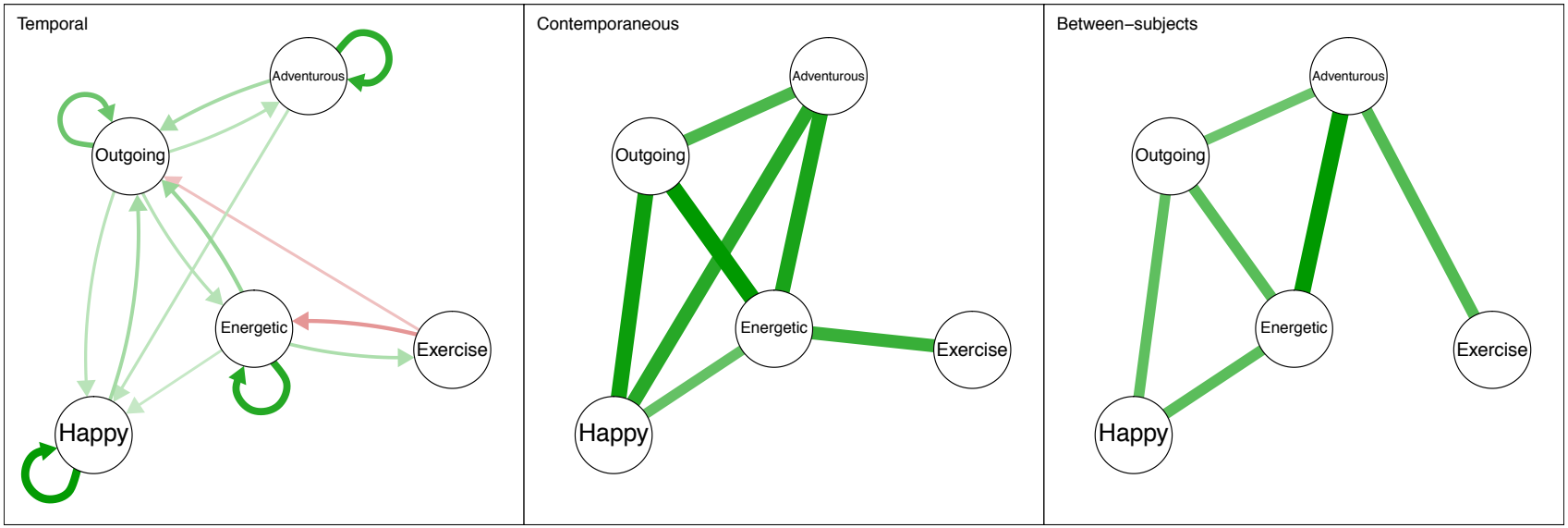
Hypothetical example of networks based on two persons:

- Clinically depressed person constantly scoring high on both
- Healthy person constantly scoring low on both

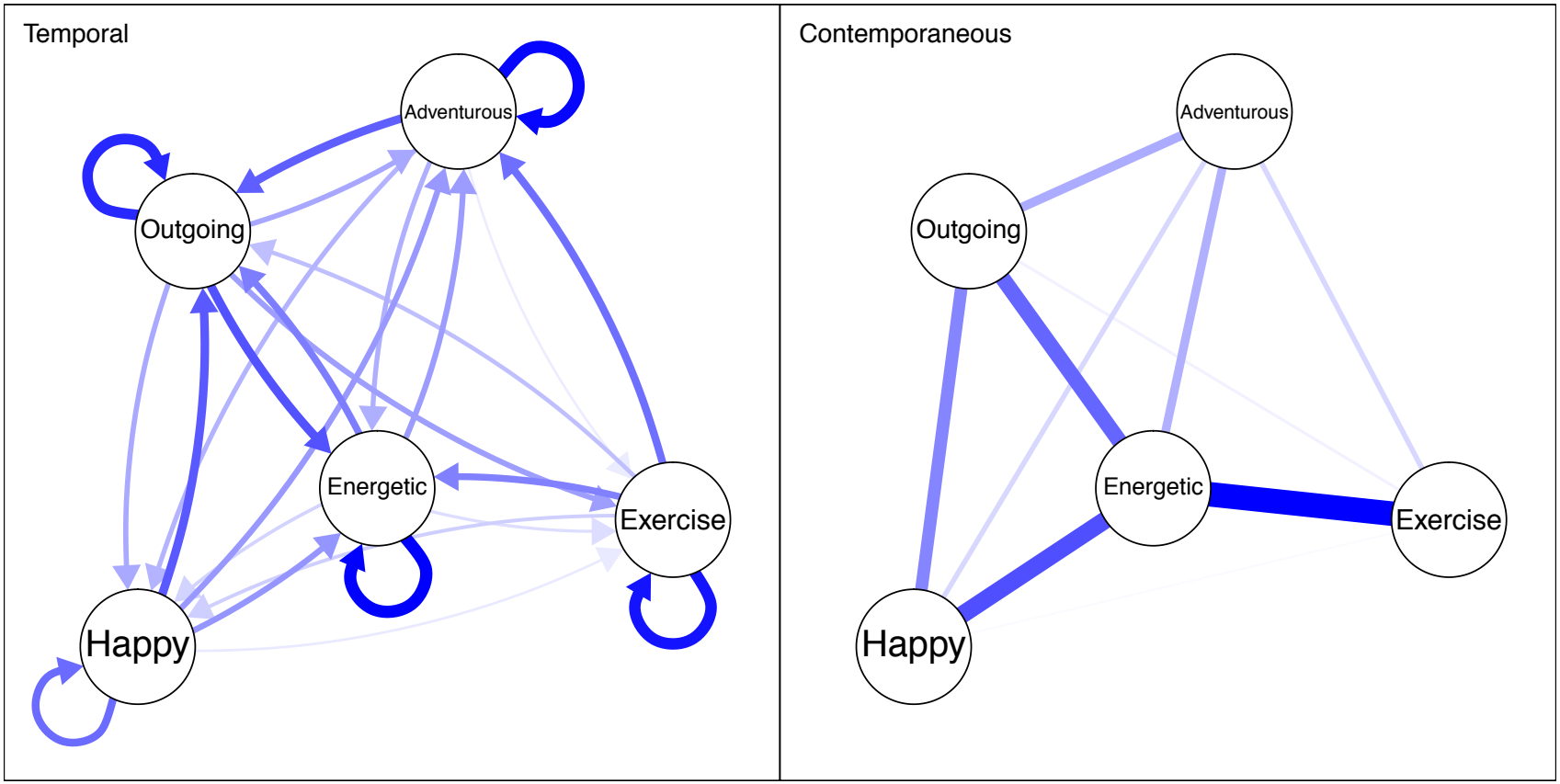
Empirical Example

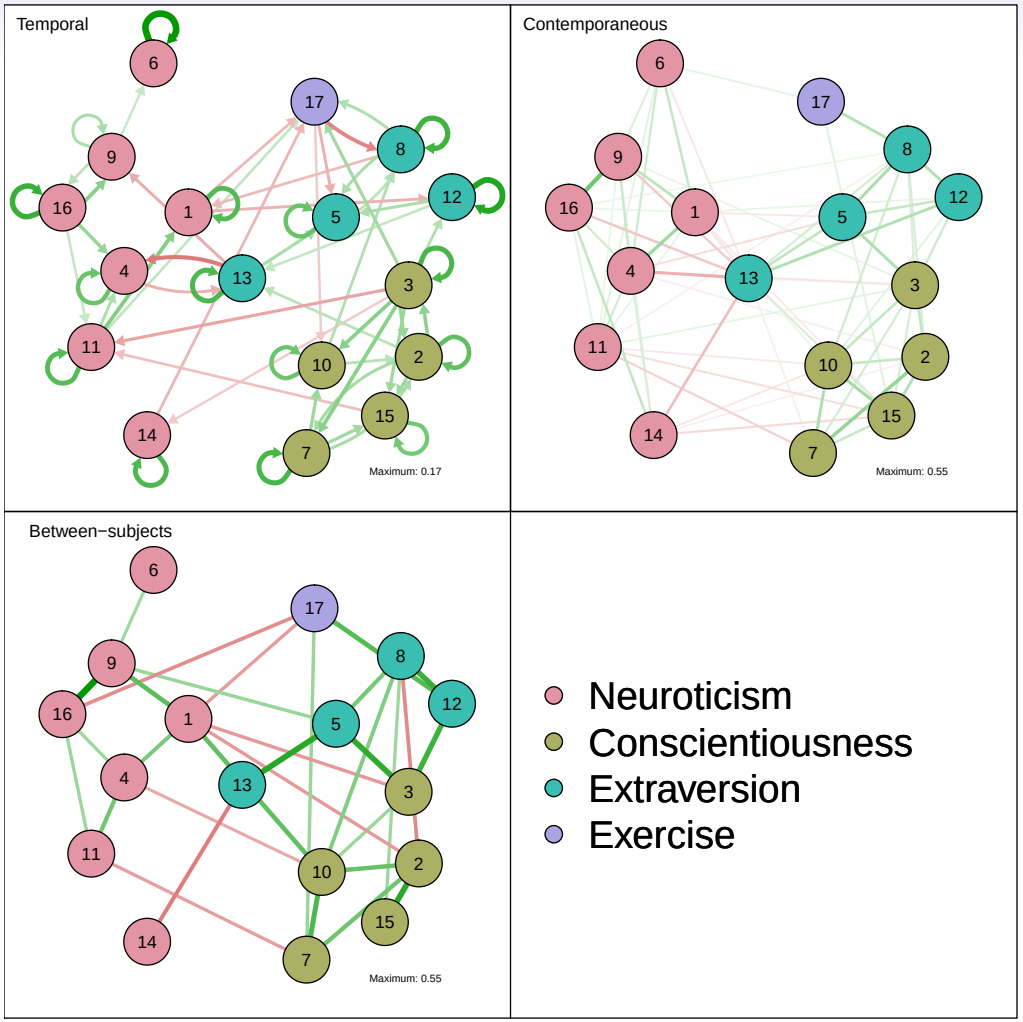
- Two datasets
  - Original: 26 subjects, 51 measurements on average, 1323 total observations
  - Replication: 65 subjects, 35.5 measurements on average, 2309 total observations
- 16 indicators of neuroticism, extroversion, conscientiousness
- Orthogonal estimation of temporal and contemporaneous effects
- Only significant effects shown
  - Alpha = 0.05 and using the "or" rule

Time series: $N > 1$

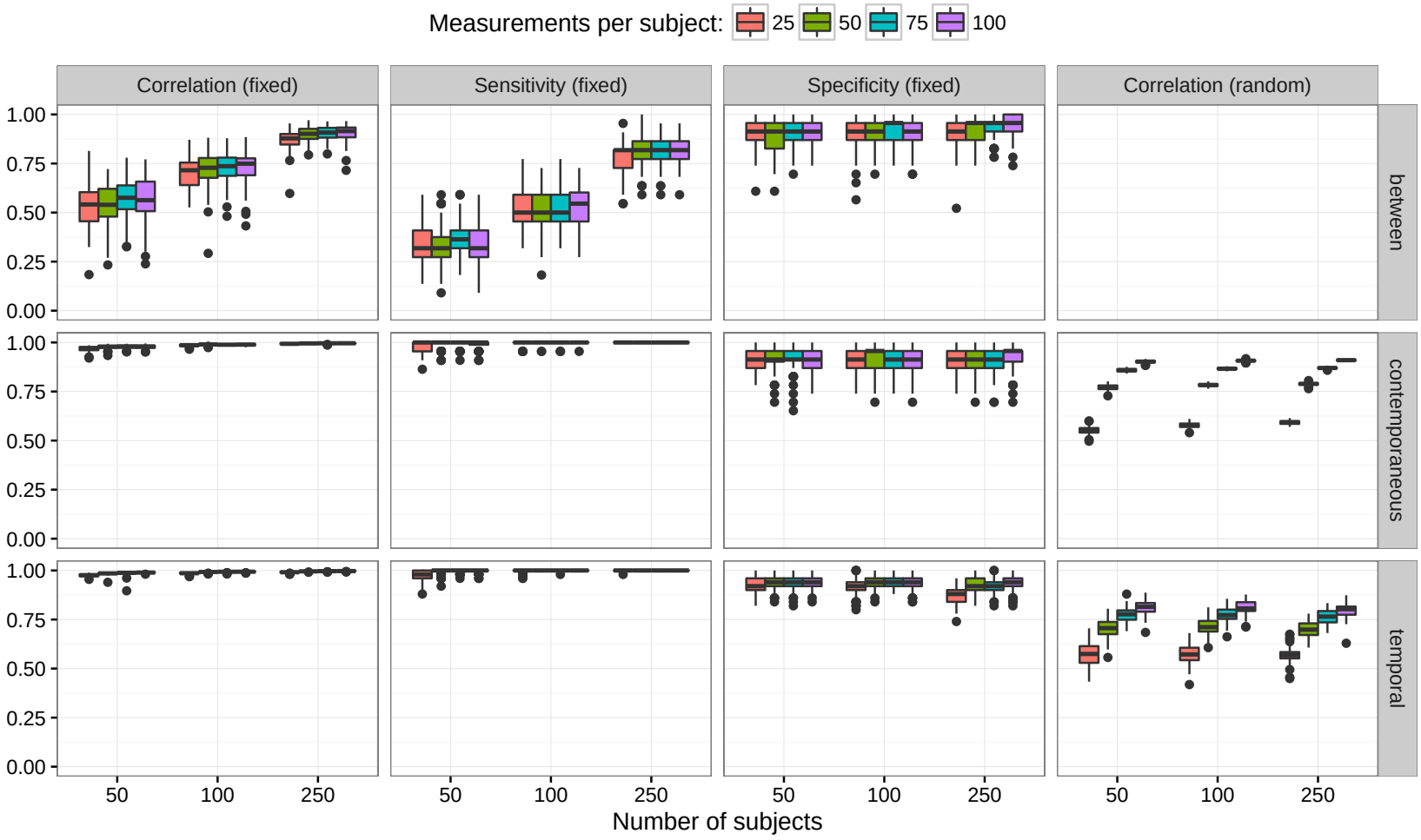


Individual Differences





1 = "Worried"; 2 = "Organized"; 3 = "Ambitious"; 4 = "Depressed"; 5 = "Outgoing"; 6 = "Self-Conscious"; 7 = "Self-Disciplined"; 8 = "Energetic"; 9 = "Frustrated"; 10 = "Focused"; 11 = "Guilty"; 12 = "Adventurous"; 13 = "Happy"; 14 = "Control"; 15 = "Achieved"; 16 = "Angry"; 17 = "Exercise."



8 variables; 50% sparse; 100 replications in each condition.

arXiv.org > stat > arXiv:1609.04156

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Statistics > Methodology

Discovering Psychological Dynamics: The Gaussian Graphical Model in Cross-sectional and Time-series Data

Sacha Epskamp, Lourens J. Waldorp, René Mõttus, Denny Borsboom

(Submitted on 14 Sep 2016 (v1), last revised 5 Oct 2016 (this version, v2))

This paper outlines statistical network models in cross-sectional and time-series data, that attempt to highlight potential causal relationships between observed variables. The paper describes three kinds of datasets. In cross-sectional data (1), one can estimate a Gaussian graphical model (GGM; a network of partial correlation coefficients). In single-subject time-series analysis (2), networks are typically constructed through the use of (multilevel) vector autoregression (VAR). VAR estimates a directed network that encodes temporal predictive effects---the temporal network. We show that GGM and VAR models are closely related: VAR generalizes the GGM by taking violations of independence between consecutive cases into account. VAR analyses can also return a GGM that encodes relationships within the same window of measurement---the contemporaneous network. When multiple subjects are measured (3), multilevel VAR estimates fixed and random temporal networks. We show that between-subject effects can also be obtained in a GGM network---the between-subjects network. We propose a novel two-step multilevel estimation procedure to obtain fixed and random effects for contemporaneous network structures. This procedure is implemented in the R package mlVAR. The paper presents a simulation study to show the performance of mlVAR and showcases the method in an empirical example on personality inventory items and physical exercise.

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Conclusion

Conclusion

- Network structures are useful in discovering potential causal relationships
- Cross-sectional data:
  - Gaussian graphical model (GGM)
- Time-series data:
  - Contemporaneous network (GGM)
  - Temporal network (VAR)
  - Between-subjects network (GGM)

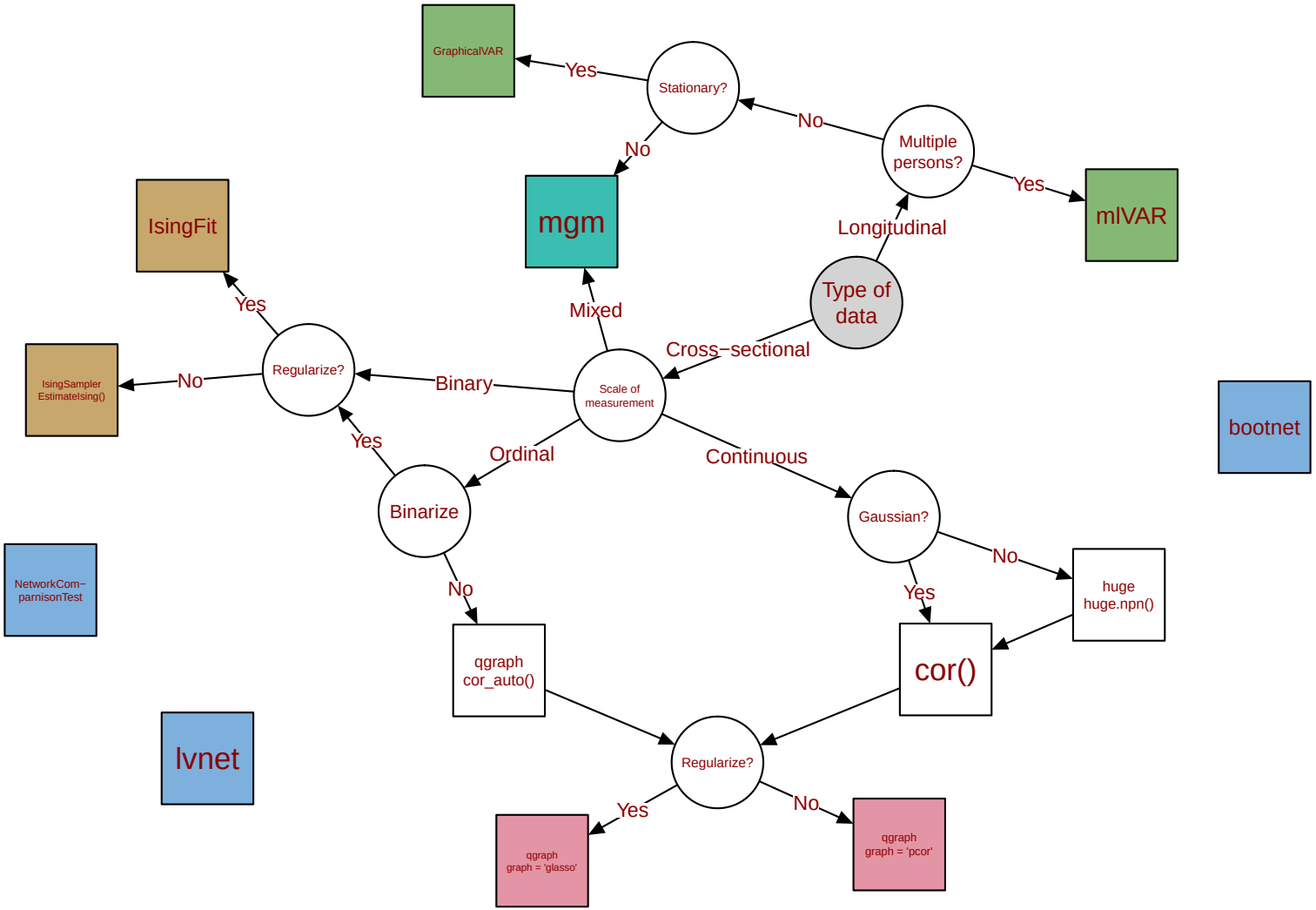
Limitations and Future Directions

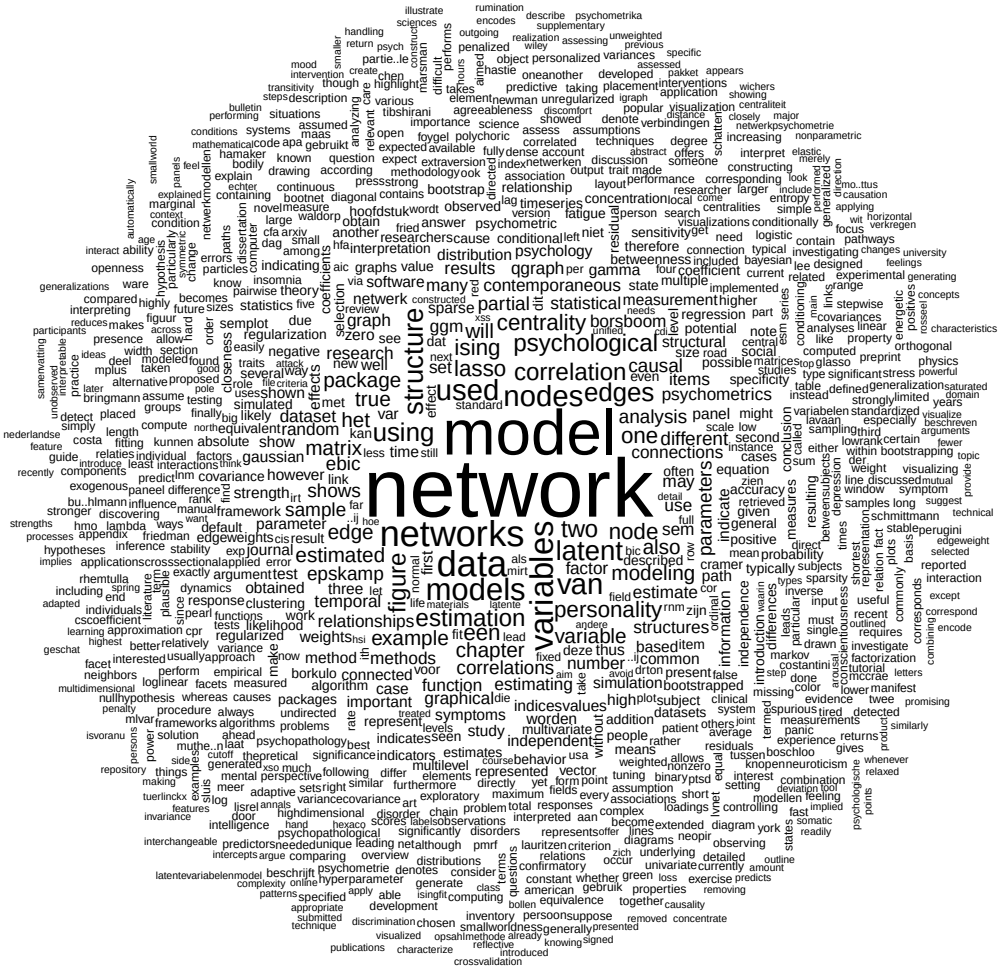
- A lot of potential problems with multi-level estimation
  - Multivariate estimation
  - Modeling random contemporaneous effects
  - Parameter variance-covariances
  - Model selection
- Possibly move away from multi-level
  - LASSO variants?
- Lag-interval

The Limit of Observational Data

- Network structures are only hypothesis generating
  - Highlighting potential causal pathways
- Observational data can *never* confirm causality
  - Mixture of experimental and observational data needed
- We need to completely rethink the modeling framework to do so

The Psychosystems Ecosystem





`http://sachaepskamp.com/` and `http://psychosystems.org/`

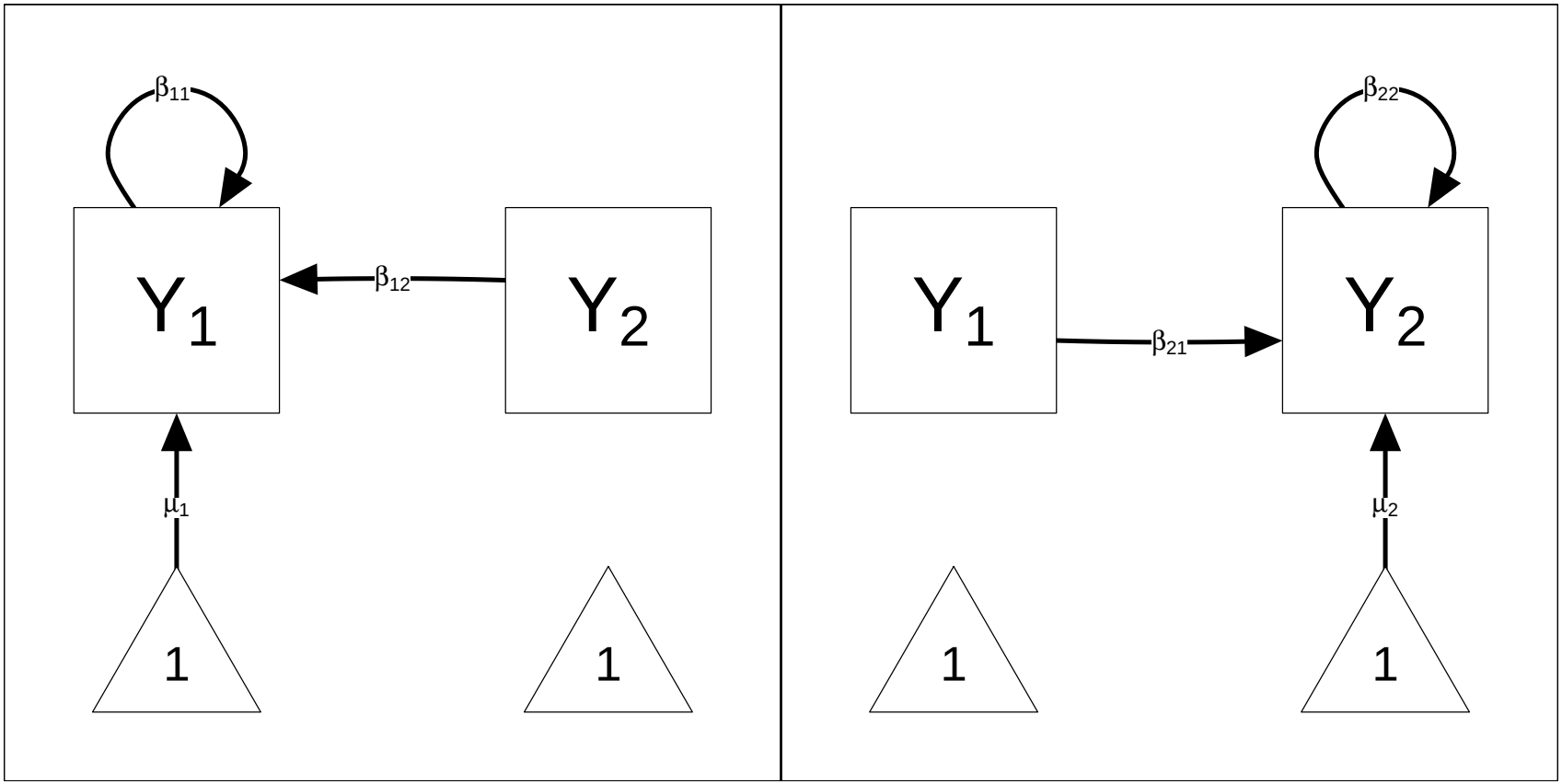
Acknowledgements

Special thanks to Hariëtte Riesse, Laura Bringmann, Noémi Schuurman and Ellen Hamaker for collaboration, helpful tips, and invigorating discussion.

Thank you for your attention!

Temporal Estimation

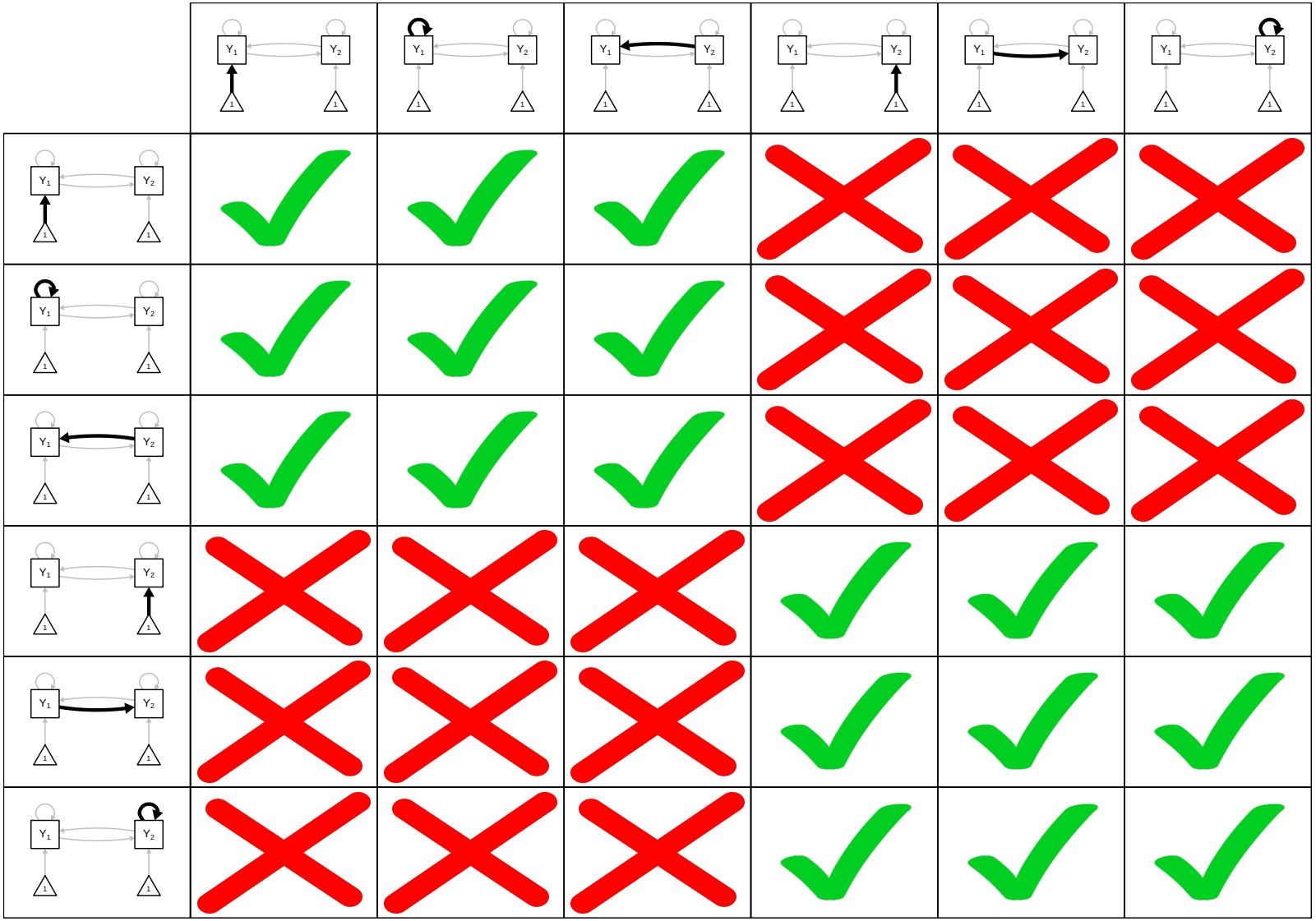
- Multi-variate multi-level MLE regression estimation is complicated and not yet well implemented in open source software
- `lme4` packages implements univariate multi-level regression
  - Douglas Bates, Martin Maechler, Ben Bolker, Steve Walker (2015). Fitting Linear Mixed-Effects Models Using lme4. Journal of Statistical Software, 67(1), 1-48. `doi:10.18637/jss.v067.i01`
  - `lmer` function
- A multi-level VAR model can be estimated by sequentially estimating univariate models
  - Estimate all incoming edges per node
  - Bringmann et al. (2013). A network approach to psychopathology: new insights into clinical time-series data. PloS one, 8(4), e60188.


β11
Y1
β12
Y2
μ1
1
1
β22
Y1
β21
Y2
μ2
1
1

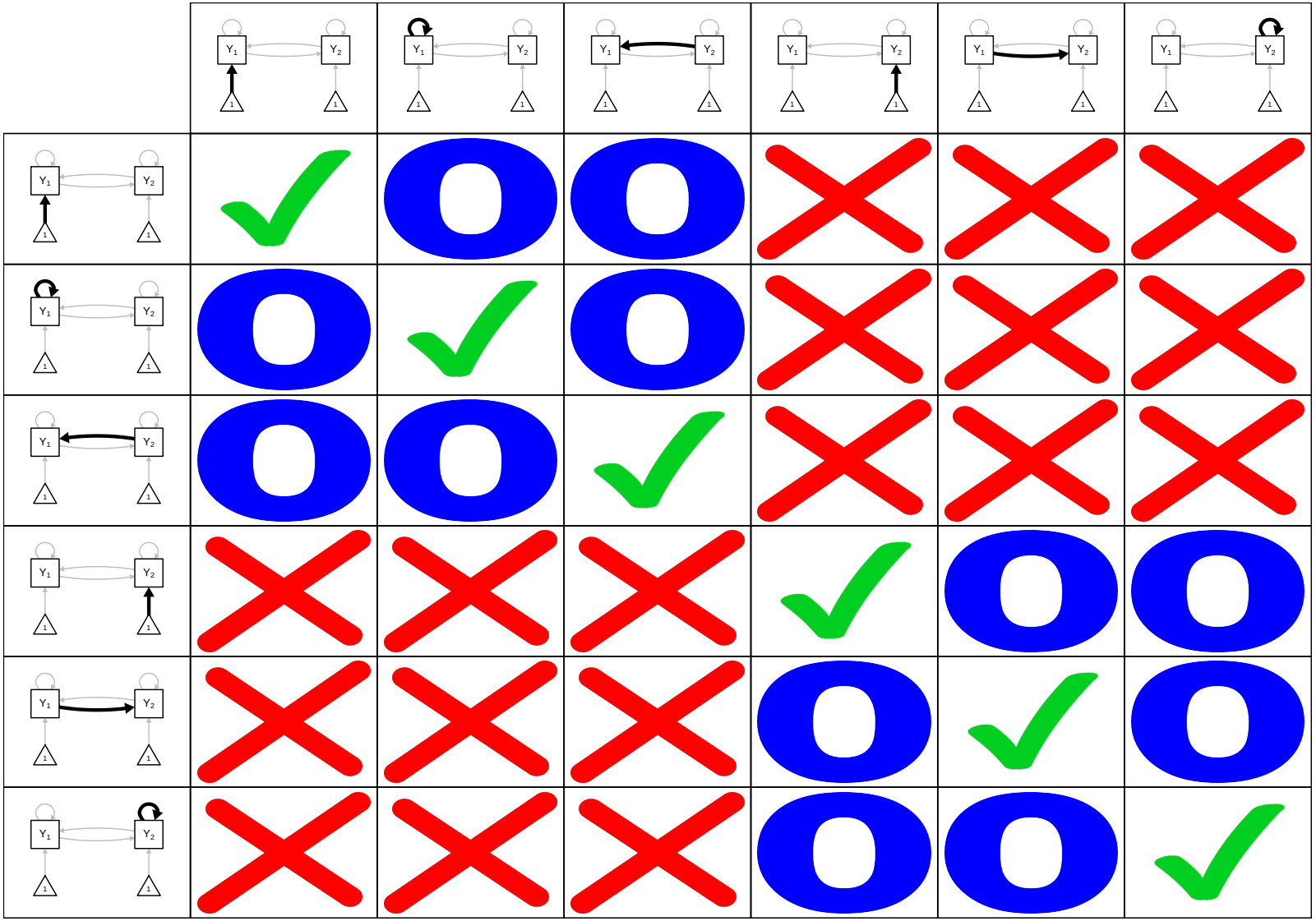
Temporal Estimation

- Correlated estimation:
  - Needs to integrate out a high-dimensional distribution over parameters
  - Only feasible for up to ~6 nodes
  - Does not estimate all parameter covariances
    - Not all parameters together in the same model
- Orthogonal estimation
  - Alternatively, parameter covariances can be fixed to zero
  - Fast, and works for high dimensions (e.g., 20 nodes)
  - But, does not return *any* parameter correlation

Correlated Estimation



Orthogonal Estimation



Between-subject Estimation

- Between subject effects can be obtained by centering predictors and adding the person-means as level 2 predictors
  - Hamaker, E. L., & Grasman, R. P. (2015). To center or not to center? Investigating inertia with a multilevel autoregressive model. Frontiers in psychology, 5, 1492.
- This can be seen as node-wise estimation of a GGM
- Thus, an estimate for the between-subjects GGM can be obtained by averaging the level-2 predictive effects standardized with the residual variances

Contemporaneous Estimation

- Contemporaneous networks need to be estimated post-hoc by investigating the residuals
- Either inverting the sample variance-covariance matrix of residuals:
  - Fixed
  - Unique
- Or as a second multi-level model using nodewise estimation of a GGM:
  - Correlated
  - Orthogonal