Network Psychometrics
Current State and Future Directions

Sacha Epskamp

IMPS 2018
The Network Perspective

The Network Perspective

PhD project (2012 - 2016)

- Funded by NWO Research talent grant
- Supervisors:
  - Promoter: Denny Borsboom
  - Co-promoter: Lourens Waldorp
- Important collaborators:
  - Gunter Maris, Mijke Rhemtulla, Eiko Fried, Claudia van Borkulo, Maarten Marsman, Angelique Cramer, Harriette Riese, Date van der Veen, Giulio Costantini, Rene Mottus

http://sachaepskamp.com/Dissertation
**Agreeableness**
- A1: Am indifferent to the feelings of others.
- A2: Inquire about others' well-being.
- A3: Know how to comfort others.
- A4: Love children.
- A5: Make people feel at ease.

**Conscientiousness**
- C1: Am exacting in my work.
- C2: Continue until everything is perfect.
- C3: Do things according to a plan.
- C4: Do things in a half-way manner.
- C5: Waste my time.

**Extraversion**
- E1: Don't talk a lot.
- E2: Find it difficult to approach others.
- E3: Know how to captivate people.
- E4: Make friends easily.
- E5: Take charge.

**Neuroticism**
- N1: Get angry easily.
- N2: Get irritated easily.
- N3: Have frequent mood swings.
- N4: Often feel blue.
- N5: Panic easily.

**Openness**
- O1: Am full of ideas.
- O2: Avoid difficult reading material.
- O3: Carry the conversation to a higher level.
- O4: Spend time reflecting on things.
- O5: Will not probe deeply into a subject.
Markov Random Fields
A graph is a set $G$ consisting of two sets: $V$ (set of nodes) and $E$ (set of edges).

Markov Random Fields

$x_i \perp \perp x_j \mid x_{-ij} \iff (i, j) \not\in E$

Graphical representation: Two variables are connected if they are not conditionally independent.

Powerful characterization of joint likelihood between observed variables.
Graphs

\[ G = \{ V, E \} \]
\[ V = \{1, 2, 3\} \]
\[ E = \{(1, 2), (2, 3)\} \]

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Graphs

Markov Random Fields

$X_i \perp \perp X_j \mid X_{-(i,j)} = x_{-(i,j)} \iff (i,j) \notin E$

- Graphical representation: Two variables are connected if they are not conditionally independent
- Powerful characterization of joint likelihood between observed variables
Markov Random Fields
In MRFs, weights should indicate conditional independence: \( X_i \perp \perp X_j | X_{-i(j)} \) \( \iff \omega_{ij} = 0 \)
Furthermore, positive weights should be comparable to negative weights in strength of association.
**Weighted graphs**

A weighted graph uses a *weights matrix* to characterize the strength of connection:

$$
\Omega = \begin{bmatrix}
0 & 0.5 & 0 \\
0.5 & 0 & -0.3 \\
0 & -0.3 & 0
\end{bmatrix}
$$

Positive edges are drawn blue (or green) and negative edges red.
Weighted graphs

A weighted graph uses a *weights matrix* to characterize the strength of connection:

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Furthermore, positive weights should be comparable to negative weights in strength of association.
Markov Random Field

In a Markov random field (MRF) a joint probability distribution is formed by multiplying potential functions $\phi_C (x_C) \geq 0$ for every clique in a graph (e.g., one variable, two connected variables, a set of three connected variables):

$$\Pr (X = x) \propto \prod_{C \in \text{cl}(G)} \phi_C (x_C)$$
Markov Random Field

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$$
\Pr (\mathbf{X} = \mathbf{x}) \propto \prod_{C \in \mathcal{C}(G)} \phi_C (\mathbf{x}_C)
$$

Pairwise MRFs only include potential function up to pairwise interactions:

$$
\Pr (\mathbf{X} = \mathbf{x}) \propto \prod_i \phi_i (x_i) \prod_{\langle ij \rangle} \phi_{ij} (x_i, x_j)
$$

These are typically used in network psychometrics
Suppose random variable $X$ with realization $x$ can take two outcomes: $s_1$ and $s_2$. We can make use of a potential function to map a unique weight to each outcome:

$$
\phi(x) = \begin{cases} 
a & \text{if } x = s_1 
\end{cases} 
\begin{cases} 
b & \text{if } x = s_2 
\end{cases}
$$
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We can use this potential function to characterize a likelihood function for \( X \):

\[
\Pr(X = x) \propto \phi(x)
\]
Suppose random variable $X$ with realization $x$ can take two outcomes: $s_1$ and $s_2$. We can make use of a potential function to map a unique weight to each outcome:

$$\phi(x) = \begin{cases} 
    a & \text{if } x = s_1 \\
    1/a & \text{if } x = s_2
\end{cases}$$

We can use this potential function to characterize a likelihood function for $X$:

$$\Pr(X = x) \propto \phi(x)$$
We can add a different potential function for $X_2$:

$$\Pr(X_1 = x_1, X_2 = x_2) \propto \phi_1(x_1)\phi_2(x_2)$$
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$$\Pr (X_1 = x_1, X_2 = x_2) \propto \phi_1(x_1)\phi_2(x_2)$$

This model implies $X_1$ and $X_2$ are independent, since:

$$\Pr (X_1 = x_1, X_2 = x_2) = \Pr (X_1 = x_1) \Pr (X_2 = x_2)$$
We can add a different potential function for $X_2$:

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Two parameters (one per potential function) used to model three probabilities (4 possible outcomes): not a saturated model.
We can add a potential function to model the \textit{pairwise interaction} between $X_1$ and $X_2$:

$$\phi_{12}(x_1, x_2) = \begin{cases} 
  c & \text{if } x_1 = s_1 \land x_2 = s_1 \\
  d & \text{if } x_1 = s_2 \land x_2 = s_1 \\
  e & \text{if } x_1 = s_1 \land x_2 = s_2 \\
  f & \text{if } x_1 = s_2 \land x_2 = s_2 
\end{cases}$$
We can add a potential function to model the *pairwise interaction* between $X_1$ and $X_2$:

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  f & \text{if } x_1 = s_2 \land x_2 = s_2 
\end{cases}
$$

To obtain:

$$
\Pr(X_1 = x_1, X_2 = x_2) \propto \phi_1(x_1)\phi_2(x_2)\phi_{12}(x_1, x_2)
$$
We can add a potential function to model the *pairwise interaction* between $X_1$ and $X_2$:

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\phi_{12}(x_1, x_2) = \begin{cases} 
    c & \text{if } x_1 = s_1 \land x_2 = s_1 \\
    1/c & \text{if } x_1 = s_2 \land x_2 = s_1 \\
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    c & \text{if } x_1 = s_2 \land x_2 = s_2
\end{cases}
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To obtain:

$$
\Pr(X_1 = x_1, X_2 = x_2) \propto \phi_1(x_1)\phi_2(x_2)\phi_{12}(x_1, x_2)
$$
Let $X^\top = [X_1 \ X_2 \ X_3]$ with realization $x$. No interaction between $X_1$ and $X_3$, $\phi_{13}(x_1, x_3) = 1 \ \forall x_1, x_3$, gives:

$$\Pr (X = x) \propto \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_{12}(x_1, x_2)\phi_{23}(x_2, x_3)$$
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$X_1$ and $X_3$ are *conditionally* independent:

$$\Pr (X_{-2} = x_{-2} \mid X_2 = x_2) \propto \phi_1(x_1)\phi_{12}(x_1, x_2)\phi_3(x_3)\phi_{23}(x_2, x_3)$$

$$\propto \phi^*_1(x_1)\phi^*_3(x_3)$$
Let $\mathbf{X}^\top = [X_1 \ X_2 \ X_3]$ with realization $\mathbf{x}$. No interaction between $X_1$ and $X_3$, $\phi_{13}(x_1, x_3) = 1 \forall x_1, x_3$, gives:

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$$\propto \phi_1^*(x_1)\phi_3^*(x_3)$$

But not marginally independent:

$$\Pr (\mathbf{X}_{-(2)} = \mathbf{x}_{-(2)}) \propto \sum_{x_2} \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_{12}(x_1, x_2)\phi_{23}(x_2, x_3)$$
This Ising Model
Pairwise Markov Random Field:

\[
\Pr (X = x) \propto \prod_{i} \phi_{i} (x_{i}) \prod_{<ij>} \phi_{ij} (x_{i}, x_{j})
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Pairwise Markov Random Field:

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\Pr (\mathbf{X} = \mathbf{x}) \propto \prod_i \phi_i (x_i) \prod_{<ij>} \phi_{ij} (x_i, x_j)
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Now encode \( S_1 = 1 \) and \( S_2 = -1 \), we can define without loss of information (in binary setting):

\[
\ln \phi_i (x_i) = \tau_i x_i \\
\ln \phi_{ij} (x_i, x_j) = \omega_{ij} x_i x_j
\]
Pairwise Markov Random Field:

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\Pr(\mathbf{X} = \mathbf{x}) \propto \prod_i \phi_i(x_i) \prod_{<ij>} \phi_{ij}(x_i, x_j)
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\ln \phi_i(x_i) = \tau_i x_i
\]
\[
\ln \phi_{ij}(x_i, x_j) = \omega_{ij} x_i x_j
\]

This gives the \textit{Ising Model}:

\[
\Pr(\mathbf{X} = \mathbf{x}) \propto \exp \left( \sum_i \tau_i x_i + \sum_{<ij>} \omega_{ij} x_i x_j \right)
\]

in which \( \tau_i \) encodes the potential of \( X_i \) to be in state 1 or \(-1\) and \( \omega_{ij} \) encodes the strength of connection between \( X_i \) and \( X_j \).
The Ising Model
Estimating an Ising model

Due to the intractable normalizing constant, standard methods for estimating an Ising model (e.g., maximum likelihood) are not practical. We can, however, easily estimate conditional distributions for node $i$ given that we observe all other nodes:

$$\Pr (X_i = X_i | X_{-i} = x_{-i}) \propto \exp \left( \tau_i + \sum_{j \neq i} \omega_{ij} x_j \right) x_i$$

This is a multiple logistic regression model! The Ising model can thus be estimated by performing several multiple logistic regressions.
Estimating an Ising model

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$$

- This is a *multiple logistic regression* model!
- The Ising model can thus be estimating by performing several multiple logistic regressions
Introduction
Markov Random Fields
Ising Model
Gaussian Graphical Model
Time-series Analysis
Conclusion

ω_{12} \quad ω_{34} \quad ω_{14} \quad ω_{23}

X_1
X_2
X_3
X_4
\[
\Pr(X_1 = 1) \propto \exp (\tau_1 + \omega_{12}x_2 + \omega_{14}x_4)
\]
Pr \( X_2 = 1 \) \( \propto \exp ( \tau_2 + \omega_{12}x_1 + \omega_{23}x_3 ) \)
Pr \((X_3 = 1) \propto \exp (\tau_3 + \omega_{23}X_2 + \omega_{34}X_4)\)
\[
\text{Pr}(X_4 = 1) \propto \exp (\tau_4 + \omega_{14} X_1 + \omega_{34} X_3)
\]
Estimating an Ising model

Multivariate estimation options:

- Optimize the *pseudolikelihood*
  \[ \prod_i \Pr(X_i = X_i \mid X_{-i} = x_{-i}) \]
- Phrase the model as a loglinear model and use ML estimation
Estimating an Ising model

Multivariate estimation options:

- Optimize the *pseudolikelihood*
  \[
  \prod_i \Pr(X_i = X_i \mid X_{-i} = x_{-i})
  \]
- Phrase the model as a loglinear model and use ML estimation

Methods for choosing which edges to include:

- Model selection
- Significance thresholding
- LASSO regularization
The Ising Model & Psychometrics

- The Ising model can be shown to be mathematically equivalent to a multidimensional IRT model.

- Rank-1 clusters in the network correspond to latent variables in the MIRT model

- This equivalence allows for re-interpretation of MIRT models and network models alike
Adding (arbitrary) variance potential function \( \ln \phi_{ii}(x_i) = \omega_{ii} x_i^2 \), we can write the Ising model as:

\[
\Pr(X = x) \propto \exp \left( \tau^T x + \frac{1}{2} x^T \Omega x \right)
\]
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We can arbitrarily set the diagonal such that \( \Omega \succ 0 \), allowing us to take the eigenvalue decomposition:

\[
\Omega = Q \Lambda Q^T,
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Now, reparameterize $\tau_i = -\delta_i$ and $-2\sqrt{\frac{\lambda_j}{2}}q_{ij} = \alpha_{ij}$ to obtain:
Adding (arbitrary) variance potential function $\ln \phi_{ii}(x_i) = \omega_{ii} x_i^2$, we can write the Ising model as:

$$\Pr(X = x) \propto \exp \left( \tau^\top x + \frac{1}{2} x^\top \Omega x \right)$$

We can arbitrarily set the diagonal such that $\Omega \succ 0$, allowing us to take the eigenvalue decomposition:

$$\Omega = Q \Lambda Q^\top,$$

Now, reparameterize $\tau_i = -\delta_i$ and $-2 \sqrt{\frac{\lambda_i}{2}} q_{ij} = \alpha_{ij}$ to obtain:

$$\Pr(X = x) = \int_{-\infty}^{\infty} f(\theta) \Pr(X = x \mid \Theta = \theta) \, d\theta$$

With:

$$\Pr(X_i = x_i \mid \Theta = \theta) = \frac{\exp \left( x_i (\alpha_i^\top \theta - \delta_i) \right)}{\sum_{x_i} \exp \left( x_i (\alpha_i^\top \theta - \delta_i) \right)}$$

$$\Theta \mid X = x \sim N \left( \pm \frac{1}{2} A^\top x, \sqrt{\frac{1}{2}} I \right)$$
The Gaussian Graphical Model
Now assume $X_i \in \mathbb{R}$ $\forall i$ and the log of all potential functions are \textit{linear}, and include variance potential function $\phi_{ii}(x_i)$:

\begin{align*}
\ln \phi_i(x_i) &= \tau_i x_i \\
\ln \phi_{ij}(x_i, x_j) &= -\kappa_{ij} x_i x_j \\
\ln \phi_{ii}(x_i) &= -\kappa_{ii} x_i^2
\end{align*}

We obtain:

$$\Pr (XXX = xxx) \propto \exp (\tau\top xxx - \frac{1}{2} xxx\top KKK xxx)$$

Now let $\Sigma = KKK^{-1}$ and $\mu = \Sigma \tau$:

$$\Pr (XXX = xxx) \propto \exp \left( -\frac{1}{2} (xxx - \mu)\top \Sigma^{-1} (xxx - \mu) \right)$$

Which implies $XXX \sim \mathcal{N}(\mu, \Sigma)$!
Now assume $X_i \in \mathbb{R} \ \forall i$ and the log of all potential functions are \textit{linear}, and include variance potential function $\phi_{ii}(x_i)$:

\[
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\ln \phi_{ii}(x_i) = -\kappa_{ii} x_i^2
\]

We obtain:

\[
\Pr(X = x) \propto \exp \left( \tau^\top x - \frac{1}{2} x^\top K x \right)
\]
Now assume $X_i \in \mathbb{R} \ \forall i$ and the log of all potential functions are \textit{linear}, and include variance potential function $\phi_{ii}(x_i)$:

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\ln \phi_{ij}(x_i, x_j) = -\kappa_{ij} x_i x_j
\]
\[
\ln \phi_{ii}(x_i) = -\kappa_{ii} x_i^2
\]

We obtain:

\[
\Pr(X = x) \propto \exp \left( \tau^\top x - \frac{1}{2} x^\top K x \right)
\]

Now let $\Sigma = K^{-1}$ and $\mu = \Sigma \tau$:

\[
\Pr(X = x) \propto \exp \left( -\frac{1}{2} (x - \mu)^\top \Sigma^{-1} (x - \mu) \right)
\]

Which implies $X \sim N(\mu, \Sigma)!$
The Gaussian Graphical Model (GGM)

The precision matrix $\mathbf{K} = \mathbf{\Sigma}^{-1}$ encodes a network structure! Furthermore, this matrix can be standardized to partial correlation coefficients:

$$\text{Cor} \left( X_i, X_j \mid X_{-i,j} \right) = -\frac{K_{ij}}{\sqrt{K_{ii}K_{jj}}} = \omega_{ij}$$
The Gaussian Graphical Model (GGM)

The precision matrix $\mathbf{K} = \mathbf{\Sigma}^{-1}$ encodes a network structure! Furthermore, this matrix can be standardized to *partial correlation coefficients*:

$$\text{Cor} \left( X_i, X_j \mid X_{-(i,j)} \right) = -\frac{\kappa_{ij}}{\sqrt{\kappa_{ii}} \sqrt{\kappa_{jj}}} = \omega_{ij}$$

Combining this information, we can form a psychometric model:

$$\mathbf{\Sigma} = \Delta (I - \Omega)^{-1} \Delta$$

- $\Delta$ is a diagonal scaling matrix
- $\Omega$ is a symmetrical matrix with 0 on the diagonal and *partial correlation coefficients* on off-diagonal elements
GGM estimation

- The GGM can be fitted similarly to SEM models, allowing for model search strategies
GGM estimation

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- The conditional distribution of a GGM is a *multiple regression model*, which allows for nodewise estimation similar to Ising model estimation.
  - Edge selection via significance thresholding or regularization.

GGM estimation

- The GGM can be fitted similarly to SEM models, allowing for model search strategies
- The conditional distribution of a GGM is a *multiple regression model*, which allows for nodewise estimation similar to Ising model estimation
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- The graphical LASSO is a fast way to obtain a range of GGM models to choose the best model from
GGM estimation

- The GGM can be fitted similarly to SEM models, allowing for model search strategies.
- The conditional distribution of a GGM is a *multiple regression model*, which allows for nodewise estimation similar to Ising model estimation.
  - Edge selection via significance thresholding or regularization.
- The graphical LASSO is a fast way to obtain a range of GGM models to choose the best model from.
- Bayesian estimation methods are also promising.
Both models imply a variance-covariance matrix $\Sigma$, aimed to closely resemble the sample variance-covariance matrix $S$ with positive degrees of freedom.
- MRF is uniquely identified and well parameterized
- MRF allows for exploratory hypothesis-generating insight in possible causal structures
Implementing networks in SEM

The variance-covariance matrices in a SEM model can be modeled as a network.

\[ \Sigma = \Lambda \Psi \Lambda^T + \Theta \]
Implementing networks in SEM

The variance-covariance matrices in a SEM model can be modeled as a network.

\[ \Sigma = \Lambda \Psi \Lambda^T + \Theta \]

Residual networks:

\[ \Theta = \Delta_\Theta (I - \Omega_\Theta)^{-1} \Delta_\Theta \]
Implementing networks in SEM

The variance-covariance matrices in a SEM model can be modeled as a network.

\[
\Sigma = \Lambda \Psi \Lambda^T + \Theta
\]

Residual networks:

\[
\Theta = \Delta_{\Theta} (I - \Omega_{\Theta})^{-1} \Delta_{\Theta}
\]

Latent networks:

\[
\Psi = \Delta_{\Psi} (I - \Omega_{\Psi})^{-1} \Delta_{\Psi}
\]
Residual Network Modeling (RNM)

$$\Sigma = \Lambda \Psi \Lambda^\top + \Delta_\Theta (I - \Omega_\Theta)^{-1} \Delta_\Theta$$

- Network is formed at the residuals of SEM
- Model a network while not assuming no unobserved common causes
- Model a latent variable structure without the assumption of local independence
Latent Network Modeling (LNM)

\[ \Sigma = \Lambda \Delta_\psi (I - \Omega_\psi)^{-1} \Delta_\psi \Lambda^T + \Theta \]

- Models conditional independence relations between latent variables as a network
- Model networks between latent variables
- Exploratory search for conditional independence relationships between latents
Time-series Analysis
Graphical Vector Auto-regression (VAR)

\[ X_t \mid x_{t-1} \sim N(Bx_{t-1}, \Theta) \]

- Variables assumed centered
- \( B \) encodes the *temporal network*
  - Temporal prediction
- \( \Theta^{-1} \) encodes the *contemporaneous network*
  - GGM
- Graphical VAR model
- Contemporaneous network: conditional concentration given $t - 1$
- Temporal network: regression coefficients between $t - 1$ and $t$
Estimation

- Multivariate multiple regression on lagged variables
- Edge selection via significance thresholding, model selection (LNM) or LASSO regularization
Estimation

- Multivariate multiple regression on lagged variables
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Important assumptions

- Stationarity
  - Means
  - Network structure
- Equidistant measurements
Multi-level VAR

Individual network model per person:

\[ X_t^{(p)} | x_{t-1}^{(p)} = N \left( \mu^{(p)} + B^{(p)} \left( x_{t-1}^{(p)} - \mu^{(p)} \right) , \Theta^{(p)} \right) \]

- Temporal and Contemporaneous network per person
- Fixed effects temporal and contemporaneous effects
- Correlations of the means can be used to construct a between-subjects network
  - Partial correlation network between means
Introduction

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Outgoing

Energetic

Adventurous

Happy

Exercise

Maximum: 0.2

Temporal

Outgoing

Energetic

Adventurous

Happy

Exercise

Maximum: 0.5

Contemporaneous

Outgoing

Energetic

Adventurous

Happy

Exercise

Maximum: 0.5

Between−subjects

Outgoing

Energetic

Adventurous

Happy

Exercise

Maximum: 0.5
### Three Methods of Estimating GVAR Models With $n > 1$ Subjects

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<th>Two-step frequentist multilevel</th>
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<td>Software</td>
<td>graphicalVAR (Epskamp, 2017b); sparseTSCGM (Abegaz &amp; Wit, 2015).</td>
<td><em>MPlus</em> 8 (Muthén &amp; Muthén, 2017; Asparouhov et al., 2016); <em>mlVAR</em> (wrapper around Mplus).</td>
<td><em>mlVAR</em> (Epskamp, Deserno, &amp; Bringmann, 2017).</td>
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<tr>
<td>Estimation</td>
<td>(1) Joint multivariate LASSO estimation with EBIC model selection (Abegaz &amp; Wit, 2013) of within-subjects centered data to obtain fixed effects temporal and contemporaneous networks. (2) glasso algorithm with EBIC model selection (Foygel &amp; Drton, 2010) on sample means of subjects to obtain between-subject network. (3) Step (1) repeated for each individual dataset to obtain subject-specific networks.</td>
<td>MCMC sampling from multivariate hierarchical model (e.g., Schuurman, Grasman, &amp; Hamaker, 2016).</td>
<td>(1) Sequential univariate multilevel regression models on previous measurement (similar to Bringmann et al., 2013), with within-subject centered lagged variables as within-subjects level predictors and sample-means of all other variables as between-subjects predictor. (2) Sequential multilevel regression models using the residuals of (1): residuals of one variable are predicted by residuals of all other variables in the same measurement occasion.</td>
</tr>
</tbody>
</table>
Future Directions & Conclusion
Current state of Network Psychometrics

- Applied in over 100 empirical papers
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  - e.g., qgraph, bootnet, lvnet, graphicalVAR, mIVAR, IsingSampler, IsingFit, mgm, NetworkToolbox, JASP
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<table>
<thead>
<tr>
<th>Title</th>
<th>Speaker / Chair</th>
<th>Day</th>
<th>Time</th>
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<td>Symposium 1: Cognitive Development: Network Psychometric Approaches</td>
<td>Maarten Marsman</td>
<td>Tuesday</td>
<td>11:00 - 12:00</td>
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<tr>
<td>Symposium 3: Psychometric Developments in JASP</td>
<td>Don van den Bergh</td>
<td>Tuesday</td>
<td>13:30 - 15:00</td>
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<td>Symposium 5: Modeling Intensive Longitudinal Data: Perks and Pitfalls</td>
<td>Laura Bringmann</td>
<td>Tuesday</td>
<td>15:20 - 16:50</td>
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<td>Diagnosing Diagnostic Models: From von Neumanns Elephant to Model Equivalencies and Network Psychometrics</td>
<td>Matthias von Davier</td>
<td>Tuesday</td>
<td>17:00 - 17:50</td>
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<td>Network Psychometrics: Current State and Future Directions</td>
<td>Sacha Epskamp</td>
<td>Tuesday</td>
<td>17:50 - 18:30</td>
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<tr>
<td>Symposium 8: Non-Cognitive Psychometric Theory and Assessment</td>
<td>Gunter Maris</td>
<td>Wednesday</td>
<td>08:30 - 10:00</td>
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<tr>
<td>Equivalent Dynamic Models</td>
<td>Peter Molenaar</td>
<td>Wednesday</td>
<td>10:15 - 11:00</td>
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<tr>
<td>Symposium 9: Timely Perspectives on Dynamic Models for Time Series and Panels</td>
<td>Eva Ceulemans; Janne Adolf</td>
<td>Wednesday</td>
<td>15:00 - 16:30</td>
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<tr>
<td>Network Analysis</td>
<td>Han van der Maas</td>
<td>Wednesday</td>
<td>16:30 - 18:00</td>
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<tr>
<td>Symposium 17: Network Psychometrics II: Psychometric Extensions to Network Modeling</td>
<td>Sacha Epskamp</td>
<td>Friday</td>
<td>08:30 - 10:00</td>
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- Methodological aspects in MRF estimation
  - MRFs for ordinal data
  - Missing data handling
  - Choosing the right estimation method

- Philosophical issues
- Within- and Between-subjects analysis
- Dynamical network modeling
- Time-varying network models
- Incorporating prior knowledge
- Multi-level generalized network models (RNM & LNM)
- Building network models from theory
- Network-based adaptive assessment (See symposium on Friday)
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Thank you for your attention!

- Website: sachaepskamp.com
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  - Dissertation at: sachaepskamp.com/dissertation
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