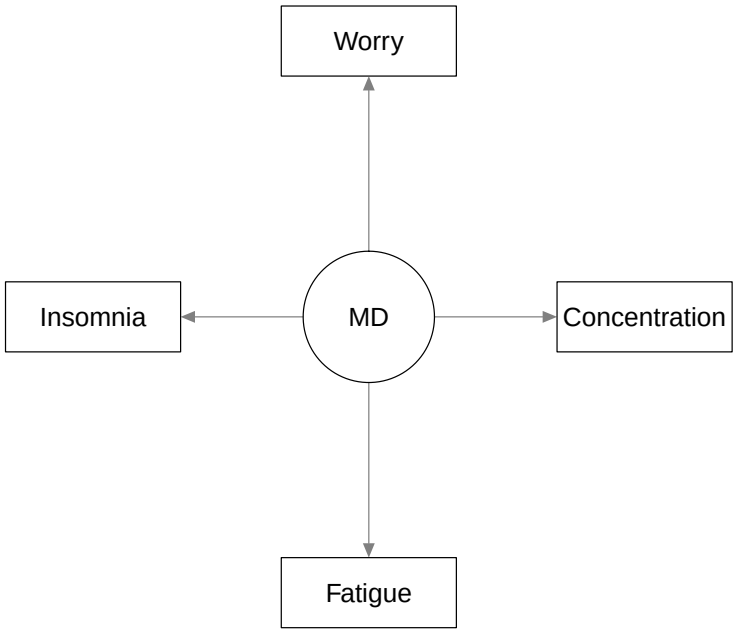
Network Psychometrics

Current State and Future Directions

Sacha Epskamp

IMPS 2018

The Network Perspective



- Cramer, A. O. J., Waldorp, L. J., van der Maas, H., & Borsboom, D. (2010). Comorbidity: A network perspective. *Behavioral and Brain Sciences, 33*, 137-193.
- Borsboom, D. (2017). A network theory of mental disorders. *World Psychiatry, 16*, 513.

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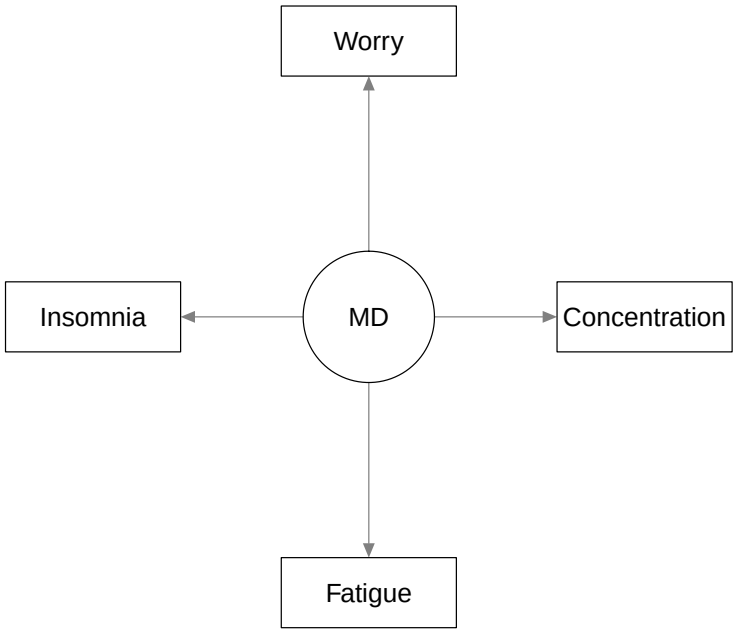

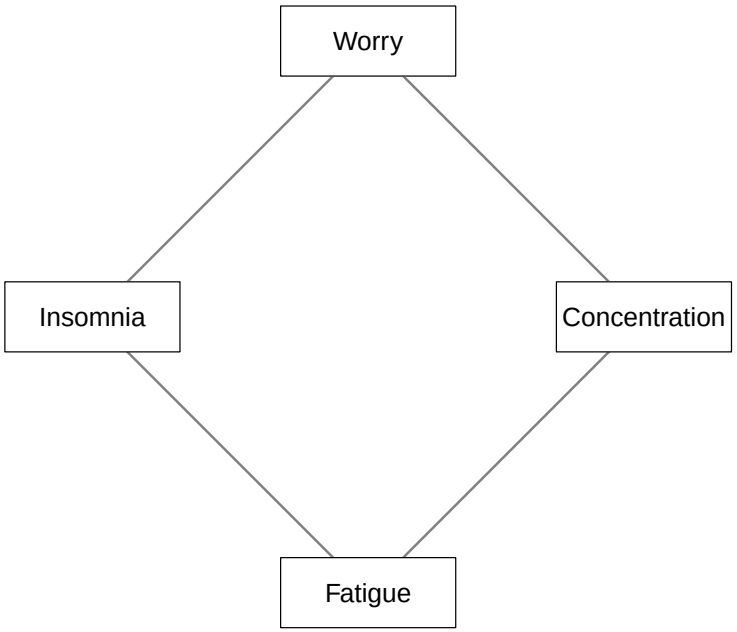

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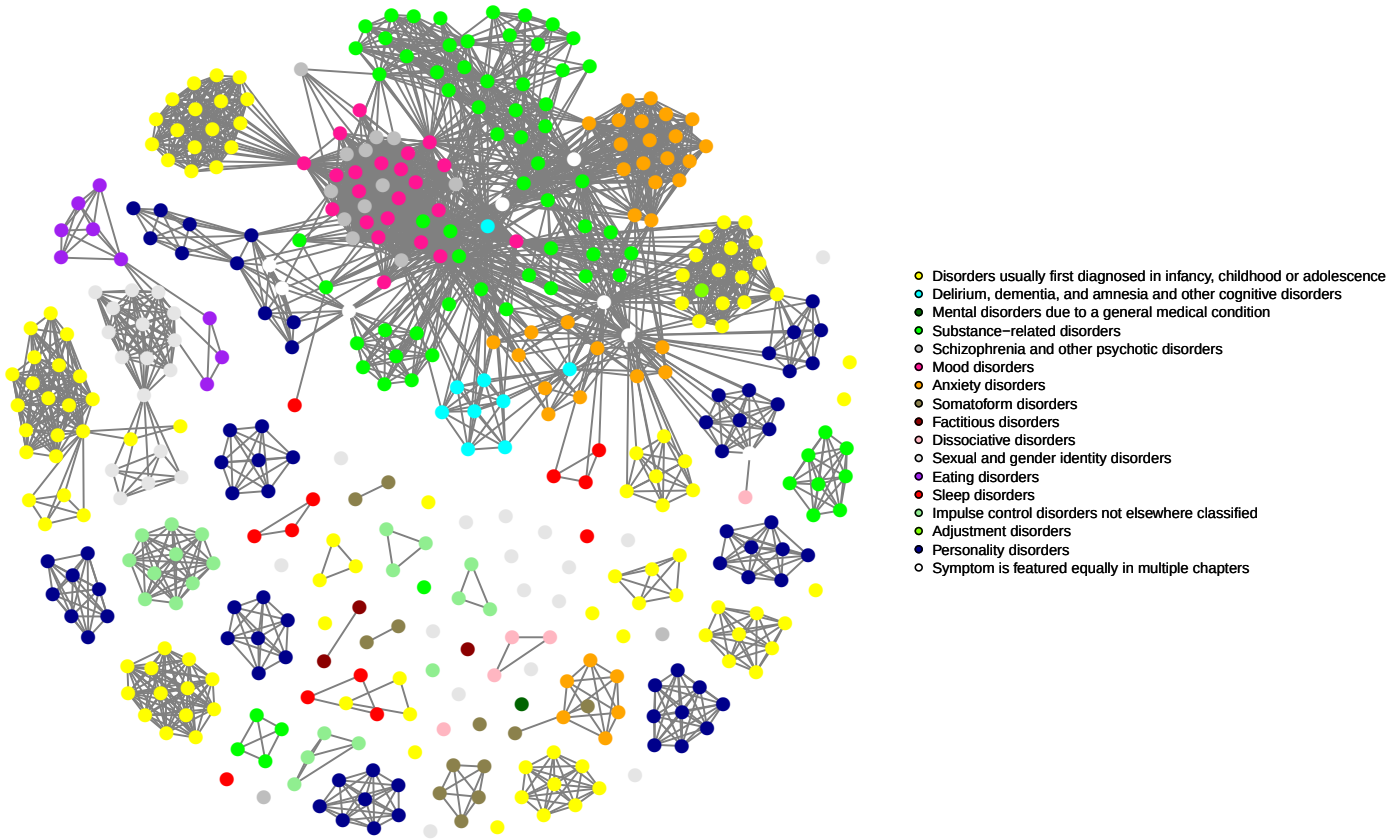
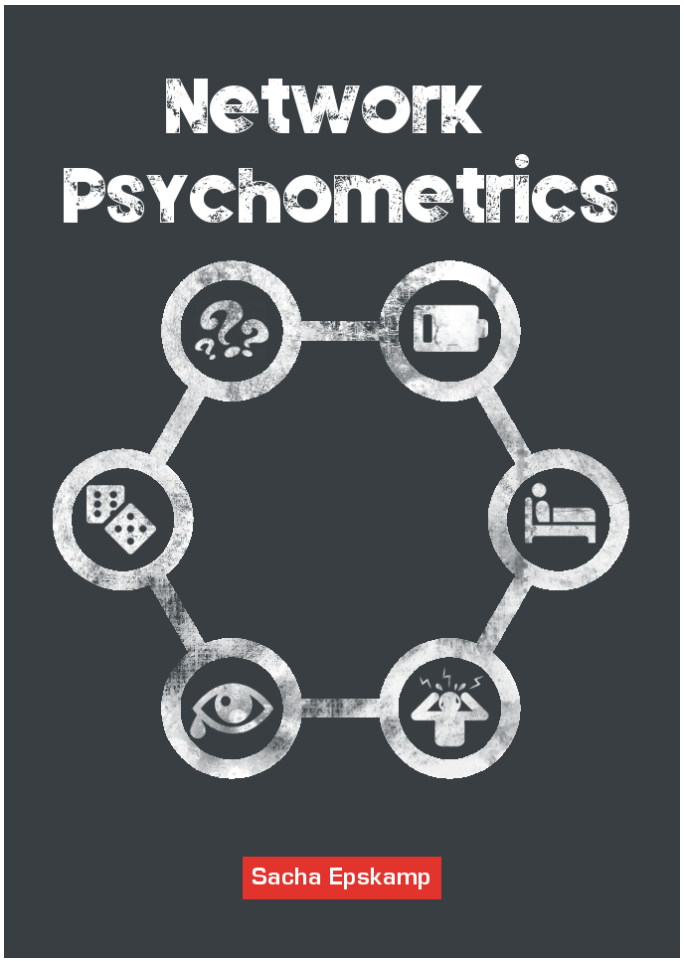


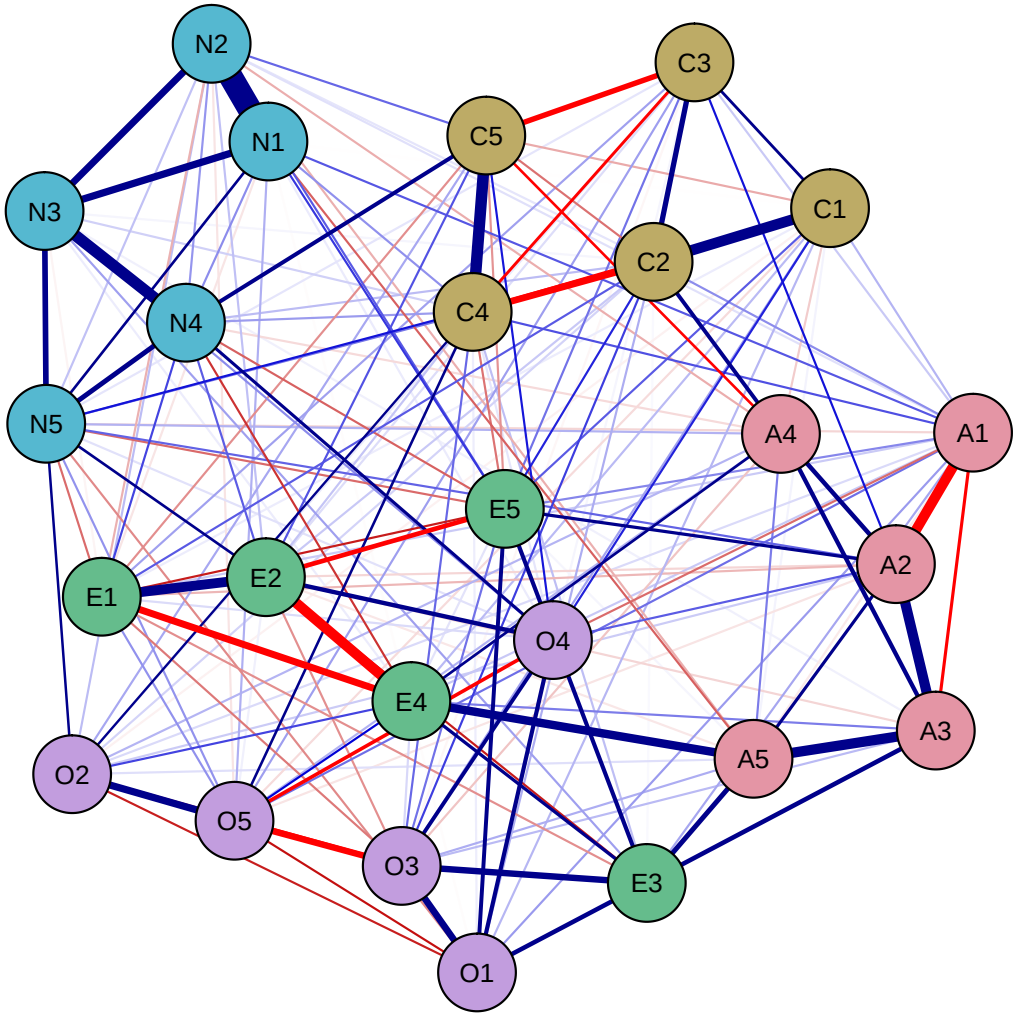
Borsboom, D., Cramer, A. O., Schmittmann, V. D., Epskamp, S., & Waldorp, L. J. (2011). The small world of psychopathology. *PloS one, 6(11)*, e27407.

PhD project (2012 - 2016)



http://sachaepskamp.com/Dissertation

- Funded by NWO Research talent grant
- Supervisors:
  - Promoter: Denny Borsboom
  - Co-promoter: Lourens Waldorp
- Important collaborators:
  - Gunter Maris, Mijke Rhemtulla, Eiko Fried, Claudia van Borkulo, Maarten Marsman, Angelique Cramer, Harriette Riese, Date van der Veen, Giulio Costantini, Rene Mottus



Agreeableness
- A1: Am indifferent to the feelings of others.
- A2: Inquire about others' well−being.
- A3: Know how to comfort others.
- A4: Love children.
- A5: Make people feel at ease.

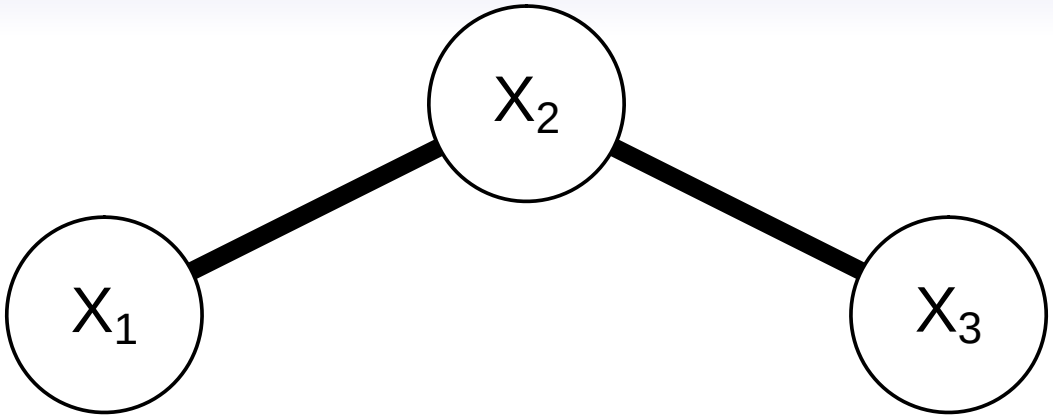
Conscientiousness
- C1: Am exacting in my work.
- C2: Continue until everything is perfect.
- C3: Do things according to a plan.
- C4: Do things in a half−way manner.
- C5: Waste my time.

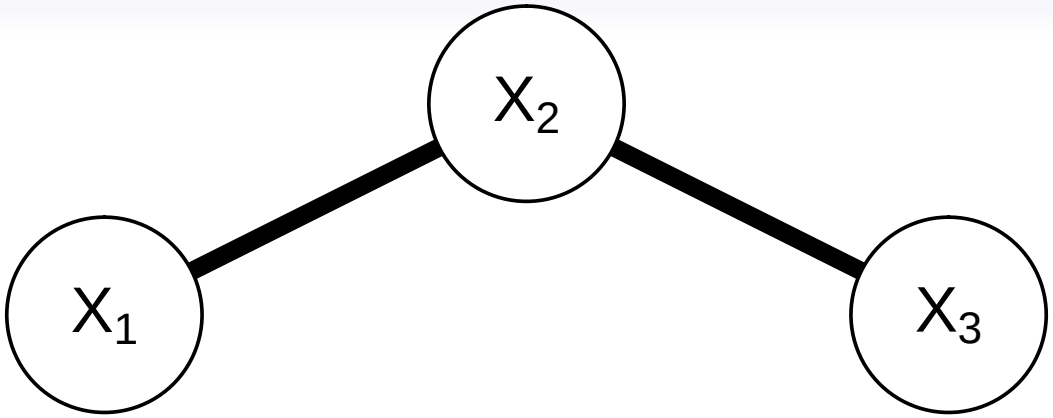
Extraversion
- E1: Don't talk a lot.
- E2: Find it difficult to approach others.
- E3: Know how to captivate people.
- E4: Make friends easily.
- E5: Take charge.

Neuroticism
- N1: Get angry easily.
- N2: Get irritated easily.
- N3: Have frequent mood swings.
- N4: Often feel blue.
- N5: Panic easily.

Openness
- O1: Am full of ideas.
- O2: Avoid difficult reading material.
- O3: Carry the conversation to a higher level.
- O4: Spend time reflecting on things.
- O5: Will not probe deeply into a subject.

Markov Random Fields

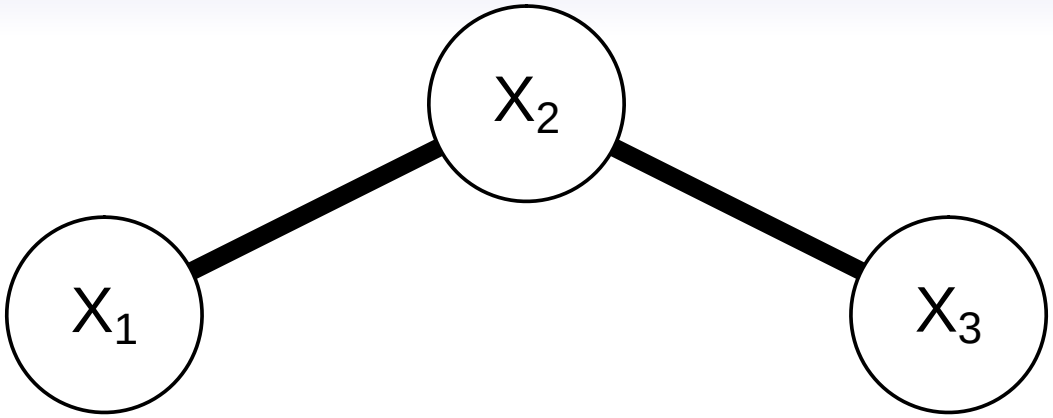

X_2
X_1
X_3



Graphs

$$G = \{V, E\}$$
$$V = \{1, 2, 3\}$$
$$E = \{(1, 2), (2, 3)\}$$

A graph is a set G consisting of two sets: V (set of nodes) and E (set of edges).



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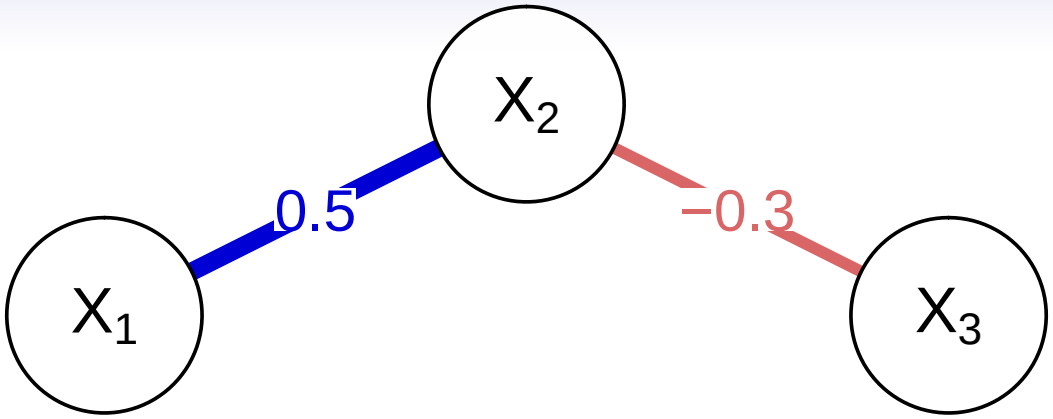
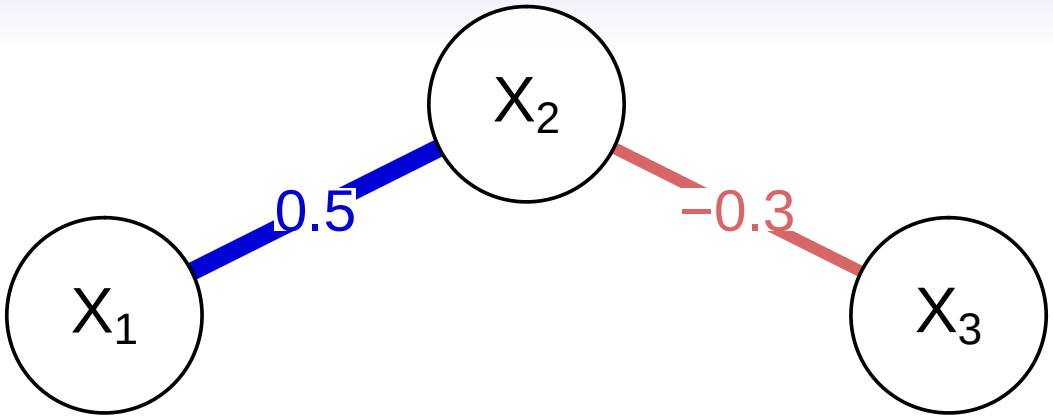
Markov Random Fields

$$X_i \perp\!\!\!\perp X_j \mid \boldsymbol{X}_{-(i,j)} = \boldsymbol{x}_{-(i,j)} \iff (i, j) \notin E$$

- Graphical representation: Two variables are connected if they are not conditionally independent
- Powerful characterization of joint likelihood between observed variables


X2
0.5
−0.3
X1
X3

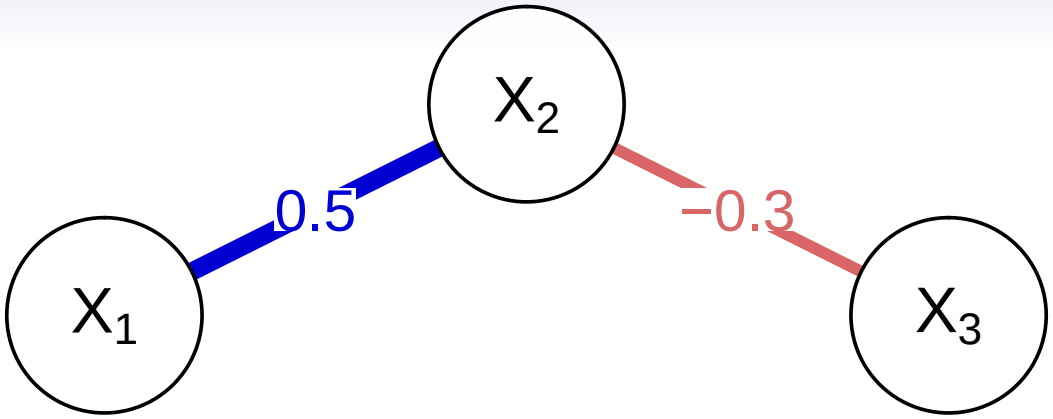


Weighted graphs

A weighted graph uses a *weights matrix* to characterize the strength of connection:

$$\mathbf{\Omega} = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & -0.3 \\ 0 & -0.3 & 0 \end{bmatrix}$$

Positive edges are drawn blue (or green) and negative edges red.



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Markov Random Fields

In MRFs, weights should indicate conditional independence:

$$X_i \perp\!\!\!\perp X_j \mid \boldsymbol{X}_{-(i,j)} = \boldsymbol{x}_{-(i,j)} \iff \omega_{ij} = 0$$

Furthermore, positive weights should be comparable to negative weights in strength of association

Markov Random Field

In a Markov random field (MRF) a joint probability distribution is formed by multiplying *potential functions* $\phi_C(\boldsymbol{x}_C) \geq 0$ for every *clique* in a graph (e.g., one variable, two connected varaibles, a set of three connected variables):

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \prod_{C \in \mathrm{cl}(G)} \phi_C(\boldsymbol{x}_C)$$

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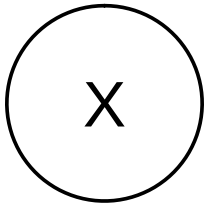
Pairwise MRFs only include potential function up to pairwise interactions:

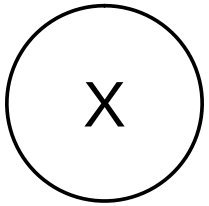
$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \prod_{i} \phi_i(x_i) \prod_{<ij>} \phi_{ij}(x_i, x_j)$$

These are typically used in network psychometrics



Suppose random variable X with realization x can take two outcomes: s_1 and s_2. We can make use of a potential function to map a unique weight to each outcome:

$$\phi(x) = \begin{cases} a & \text{if } x = s_1 \\ b & \text{if } x = s_2 \end{cases}$$



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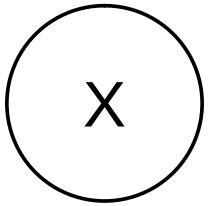
$$\Pr(X = x) \propto \phi(x)$$



Suppose random variable X with realization x can take two outcomes: s_1 and s_2. We can make use of a potential function to map a unique weight to each outcome:

$$\phi(x) = \begin{cases} a & \text{if } x = s_1 \\ 1/a & \text{if } x = s_2 \end{cases}$$

We can use this potential function to characterize a likelihood function for X:

$$\Pr(X = x) \propto \phi(x)$$

We can add a different potential function for X_2:

$$\Pr(X_1 = x_1, X_2 = x_2) \propto \phi_1(x_1)\phi_2(x_2)$$

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This model implies X_1 and X_2 are independent, since:

$$\Pr(X_1 = x_1, X_2 = x_2) = \Pr(X_1 = x_1)\Pr(X_2 = x_2)$$

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Two parameters (one per potential function) used to model three probabilities (4 possible outcomes): not a saturated model.

We can add a potential function to model the *pairwise interaction* between X_1 and X_2:

$$\phi_{12}(x_1, x_2) = \begin{cases} c & \text{if } x_1 = s_1 \wedge x_2 = s_1 \\ d & \text{if } x_1 = s_2 \wedge x_2 = s_1 \\ e & \text{if } x_1 = s_1 \wedge x_2 = s_2 \\ f & \text{if } x_1 = s_2 \wedge x_2 = s_2 \end{cases}$$

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To obtain:

$$\Pr(X_1 = x_1, X_2 = x_2) \propto \phi_1(x_1)\phi_2(x_2)\phi_{12}(x_1, x_2)$$

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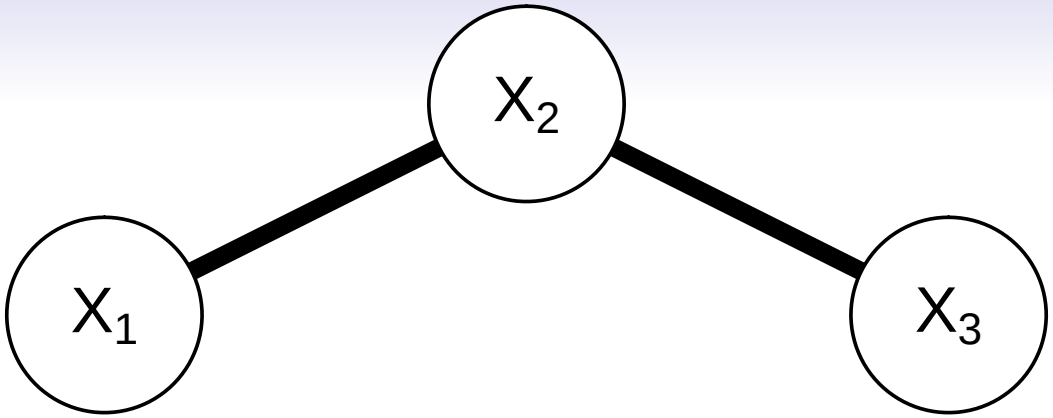
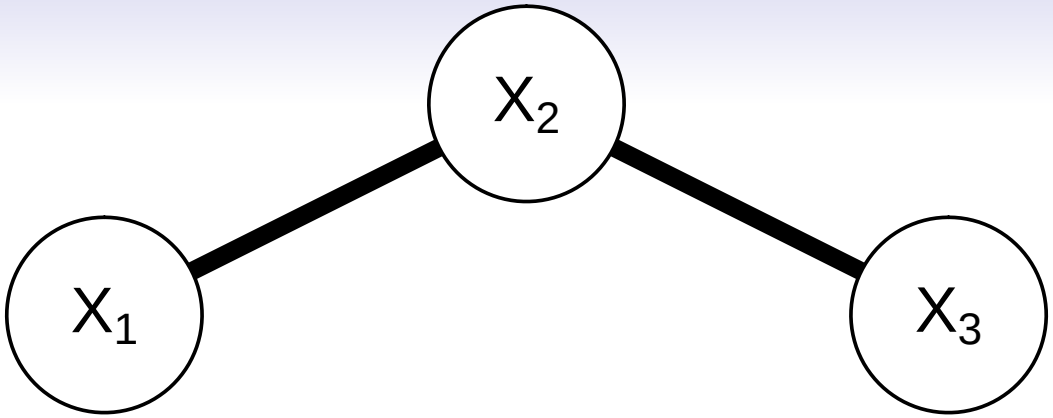


Let $\boldsymbol{X}^\top = \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix}$ with realization $\boldsymbol{x}$. No interaction between X_1 and X_3, $\phi_{13}(x_1, x_3) = 1 \, \forall_{x1,x3}$, gives:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_{12}(x_1, x_2)\phi_{23}(x_2, x_3)$$



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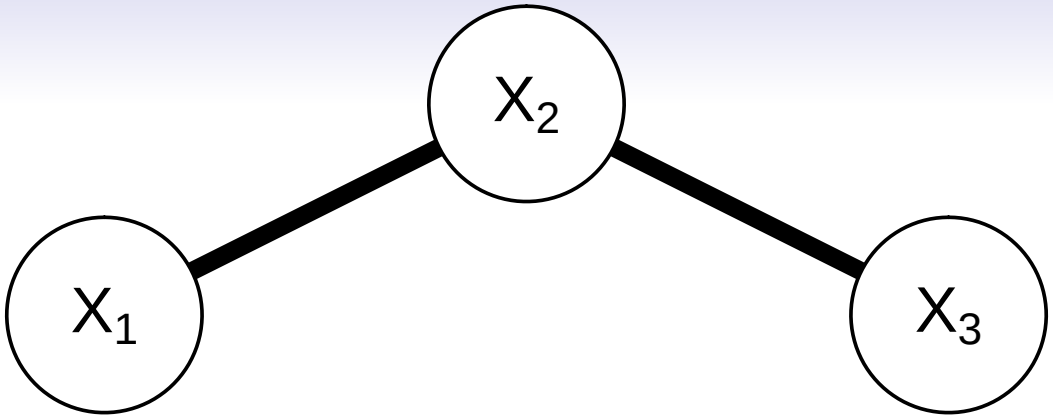
X_1 and X_3 are *conditionally* independent:

$$\begin{aligned} \Pr\left(\boldsymbol{X}_{-(2)} = \boldsymbol{x}_{-(2)} \mid X_2 = x_2\right) &\propto \phi_1(x_1)\phi_{12}(x_1, x_2)\phi_3(x_3)\phi_{23}(x_2, x_3) \\ &\propto \phi_1^*(x_1)\phi_3^*(x_3) \end{aligned}$$



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But not *marginally* independent:

$$\Pr\left(\boldsymbol{X}_{-(2)} = \boldsymbol{x}_{-(2)}\right) \propto \sum_{x_2} \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_{12}(x_1, x_2)\phi_{23}(x_2, x_3)$$

This Ising Model

Pairwise Markov Random Field:

$$\Pr\left(\boldsymbol{X} = \boldsymbol{x}\right) \propto \prod_{i} \phi_i\left(x_i\right) \prod_{<ij>} \phi_{ij}\left(x_i, x_j\right)$$

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Now encode $S_1 = 1$ and $S_2 = -1$, we can define without loss of information (in binary setting):

$$\ln \phi_i(x_i) = \tau_i x_i$$
$$\ln \phi_{ij}(x_i, x_j) = \omega_{ij} x_i x_j$$

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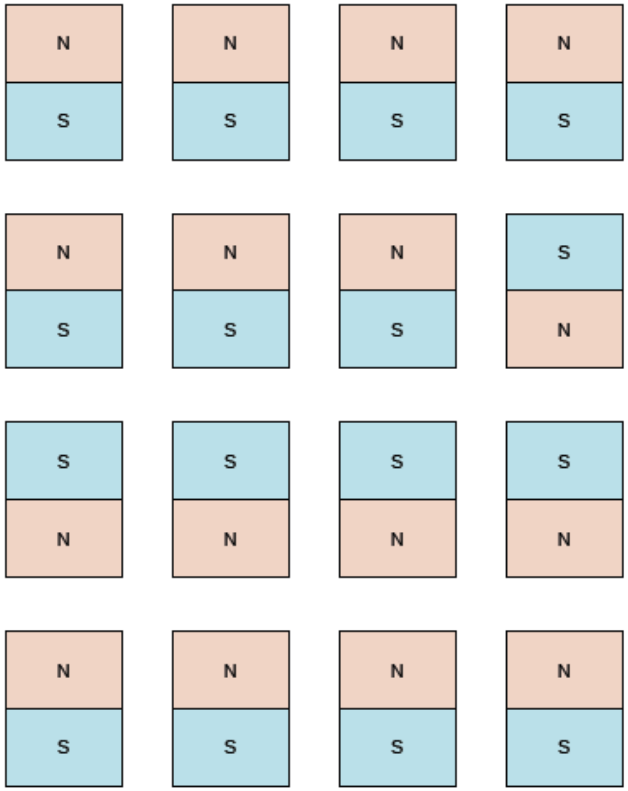
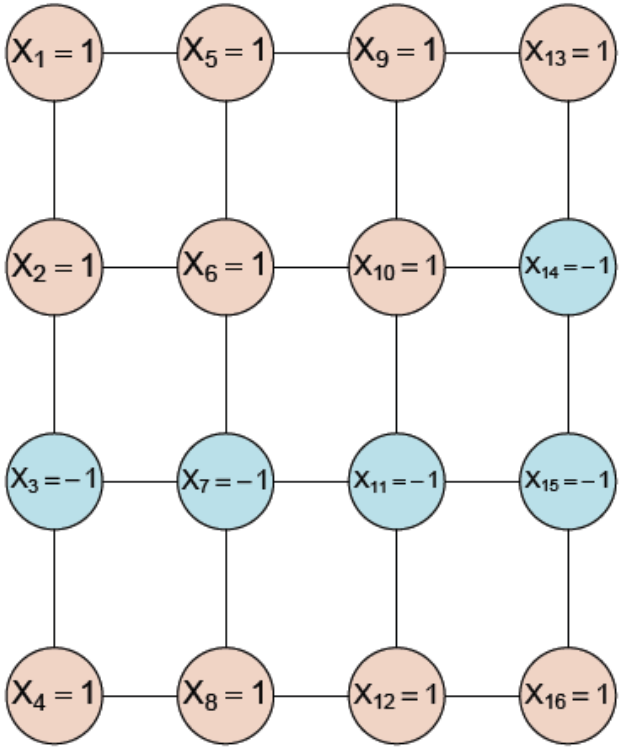
This gives the *Ising Model*:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \exp\left(\sum_{i} \tau_i x_i + \sum_{<ij>} \omega_{ij} x_i x_j\right)$$

in which τ_i encodes the potential of X_i to be in state 1 or -1 and ω_{ij} encodes the strength of connection between X_i and X_j.

The Ising Model





Estimating an Ising model

Due to the intractable normalizing constant, standard methods for estimating an Ising model (e.g., maximum likelihood) are not practical. We can, however, easily estimate conditional distributions for node i given that we observe **all** other nodes:

$$\Pr\left(X_i = X_i \mid \boldsymbol{X}_{-(i)} = \boldsymbol{x}_{-(i)}\right) \propto \exp\left(\left(\tau_i + \sum_{j, j \neq i} \omega_{ij} x_j\right) x_i\right)$$

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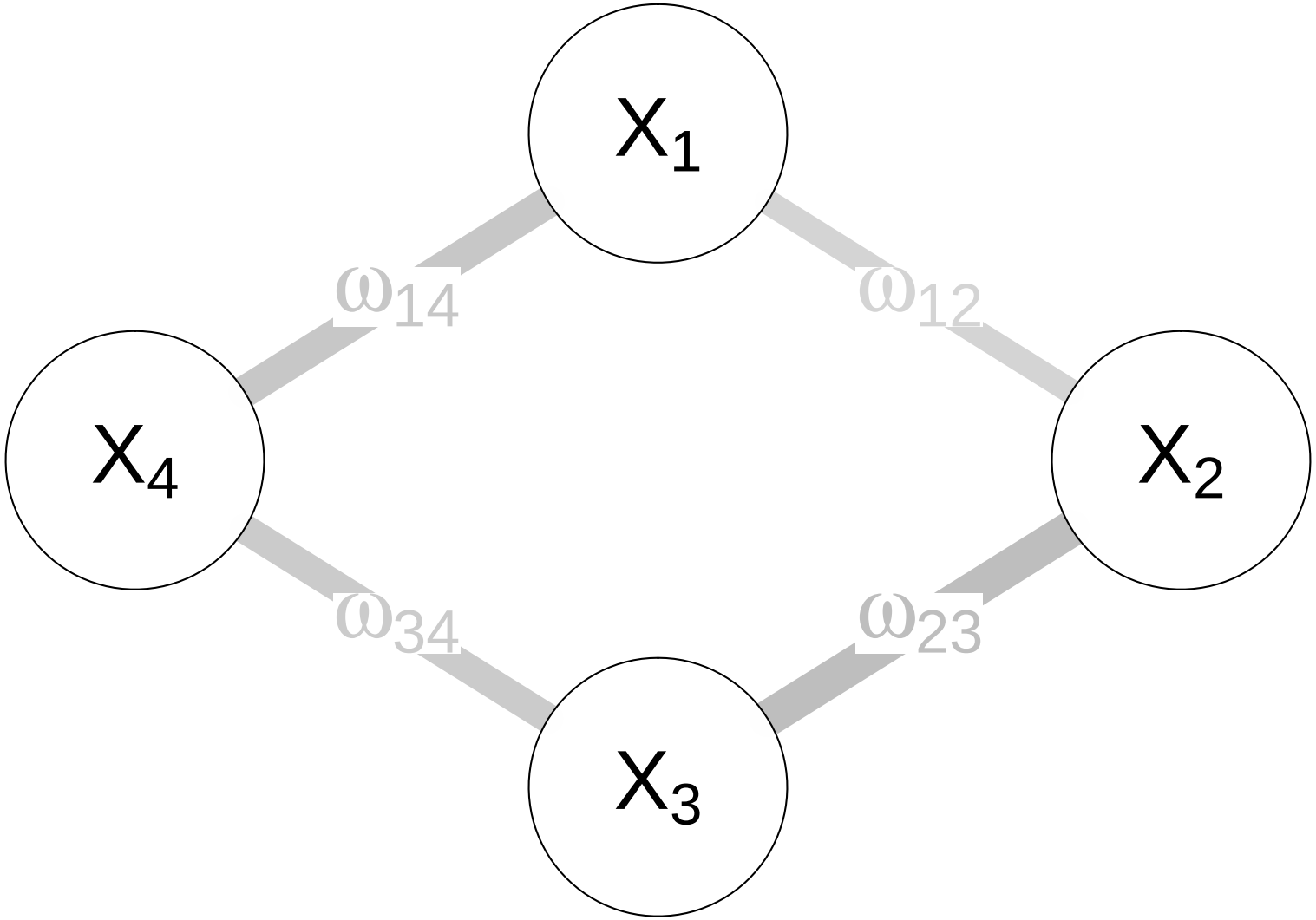
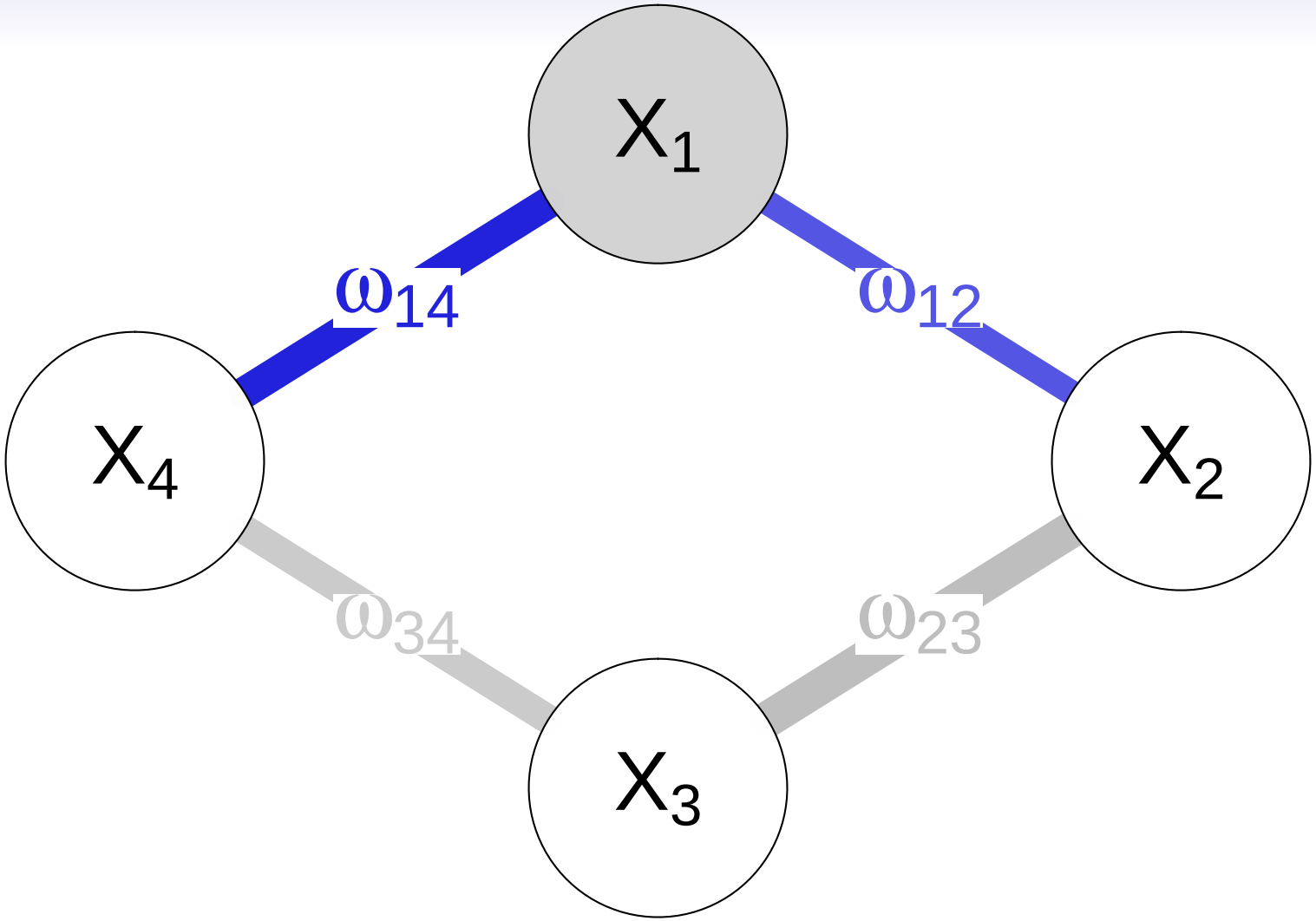
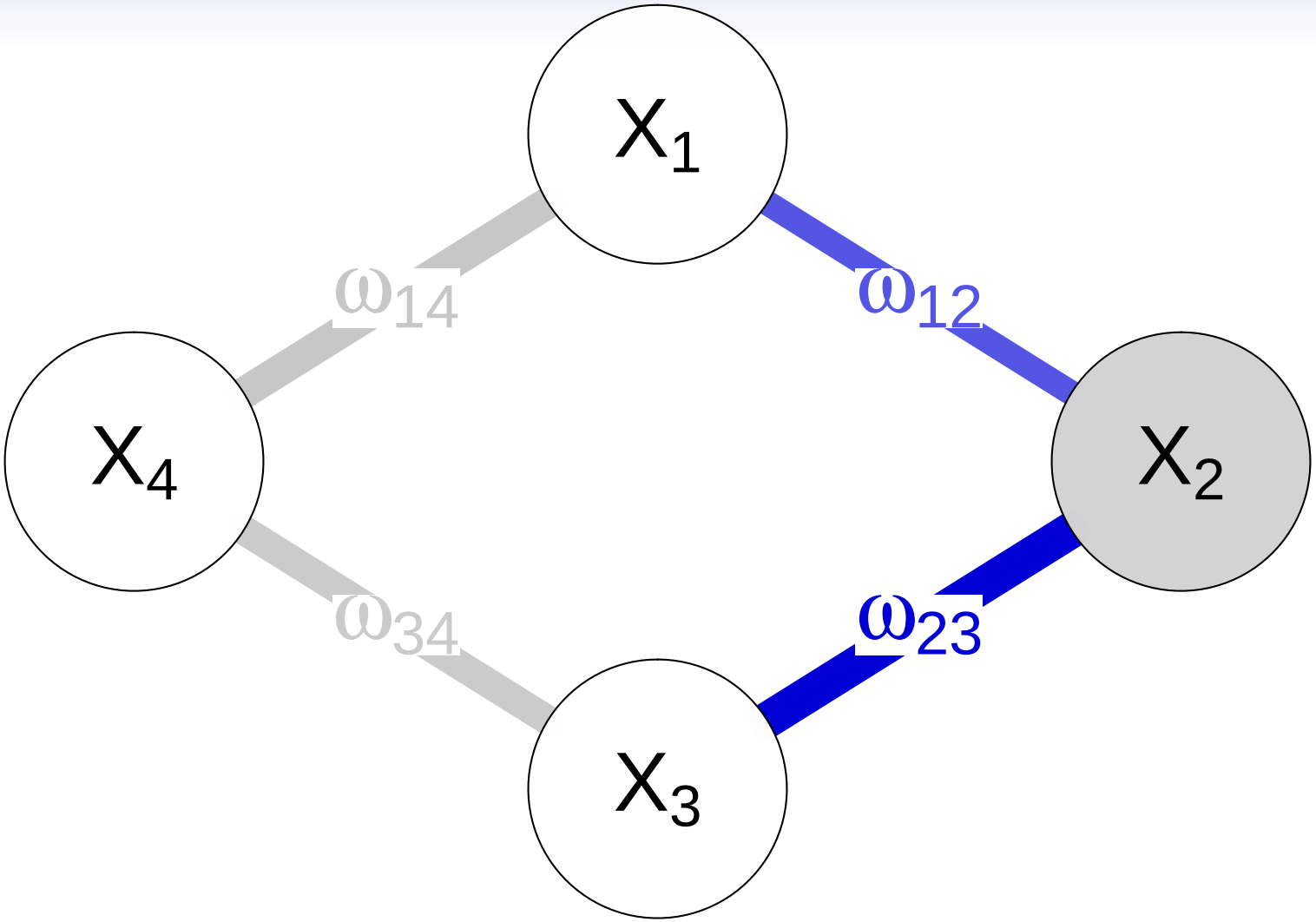
- This is a *multiple logistic regression* model!
- The Ising model can thus be estimating by performing several multiple logistic regressions

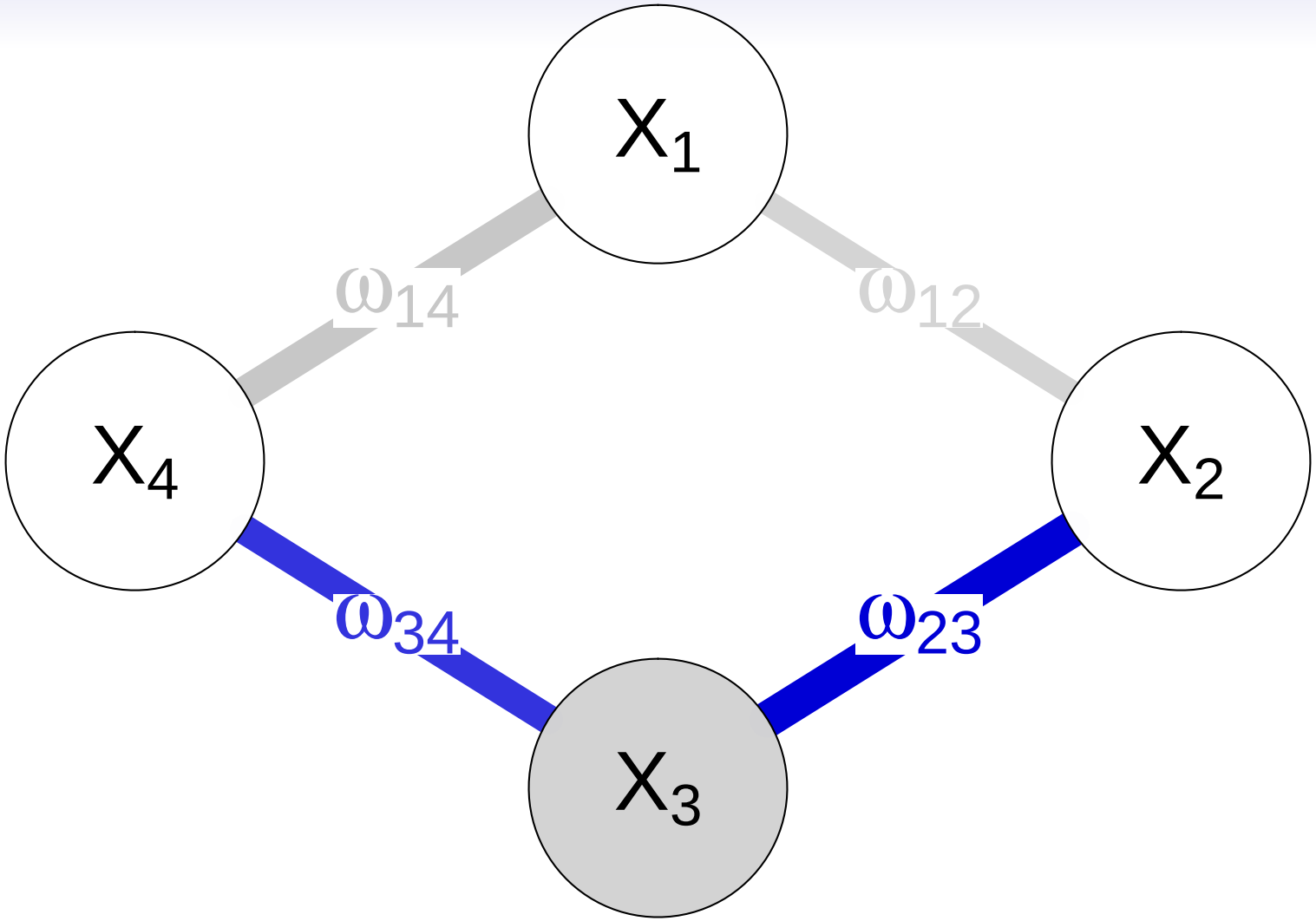

X_1
X_2
X_3
X_4
ω_{14}
ω_{12}
ω_{34}
ω_{23}



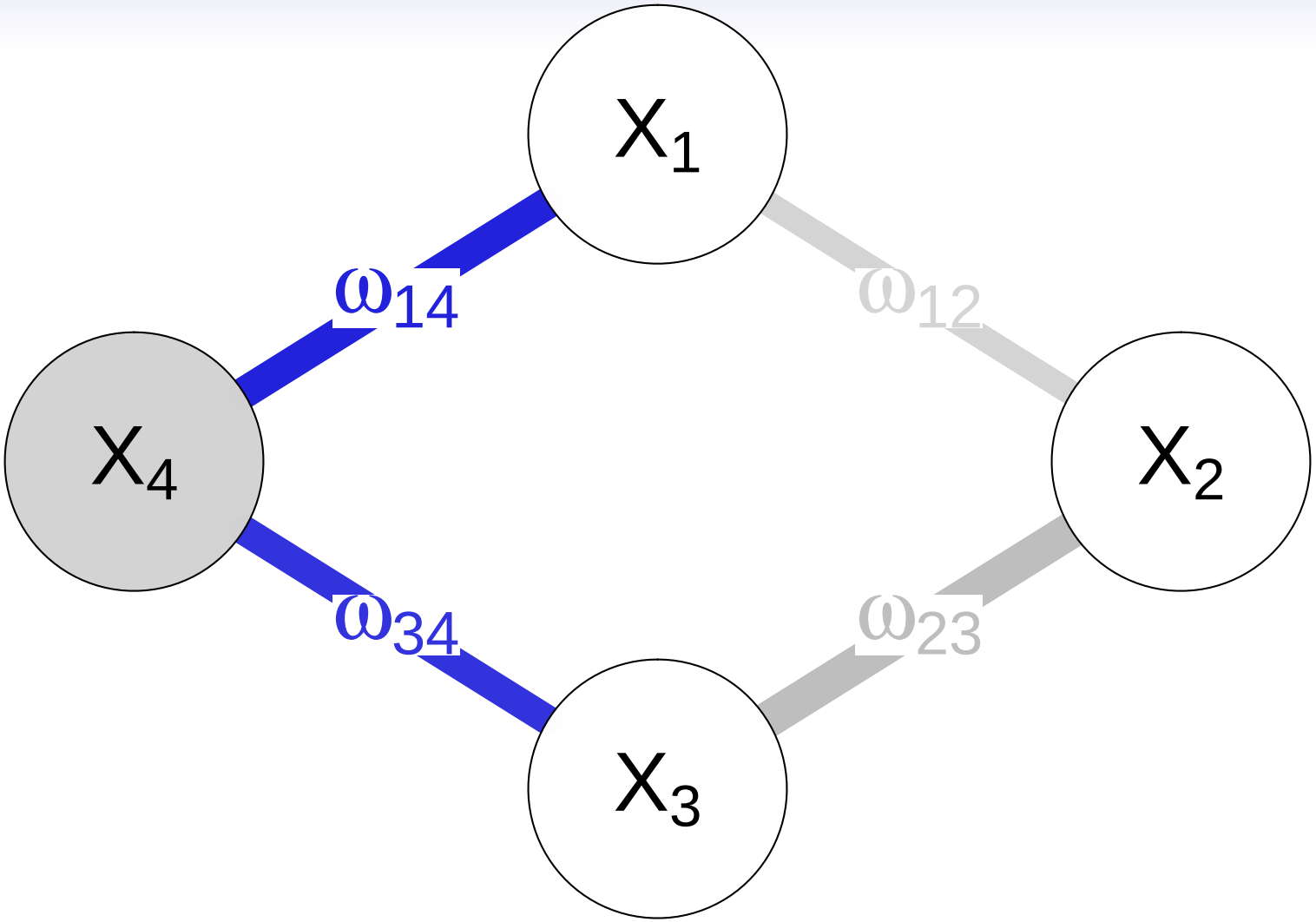
$$\Pr\left(X_1 = 1\right) \propto \exp\left(\tau_1 + \omega_{12}x_2 + \omega_{14}x_4\right)$$



$$\Pr(X_2 = 1) \propto \exp(\tau_2 + \omega_{12} x_1 + \omega_{23} x_3)$$



$$\Pr(X_3 = 1) \propto \exp(\tau_3 + \omega_{23} x_2 + \omega_{34} x_4)$$



$$\Pr\left(X_4 = 1\right) \propto \exp\left(\tau_4 + \omega_{14} x_1 + \omega_{34} x_3\right)$$

Estimating an Ising model

Multivariate estimation options:

- Optimize the *pseudolikelihood* $\prod_i \Pr\left(X_i = X_i \mid \boldsymbol{X}_{-(i)} = \boldsymbol{x}_{-(i)}\right)$
- Phrase the model as a loglinear model and use ML estimation

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Methods for choosing which edges to include:

- Model selection
- Significance thresholding
- LASSO regularization
  - van Borkulo, C. D., Borsboom, D., Epskamp, S., Blanken, T. F., Boschloo, L., Schoevers, R. A., & Waldorp, L. J. (2014). A new method for constructing networks from binary data. *Scientific reports, 4* (5918).

The Ising Model & Psychometrics

- The Ising model can be shown to be mathematically equivalent to a multidimensional IRT model.
  - e.g., Marsman, M., Borsboom, D., Kruis, J., Epskamp, S., van Bork, R., Waldorp, L. J., van der Maas, H. L. J. & **Maris, G. K. J.** (2018). An introduction to Network Psychometrics: Relating Ising network models to item response theory models. *Multivariate Behavioral Research, 53(1)*, 15-35.
- Rank-1 clusters in the network correspond to latent variables in the MIRT model
- This equivalence allows for re-interpretation of MIRT models and network models alike

Adding (arbitrary) variance potential function $\ln \phi_{ii}(x_i) = \omega_{ii} x_i^2$, we can write the Ising model as:

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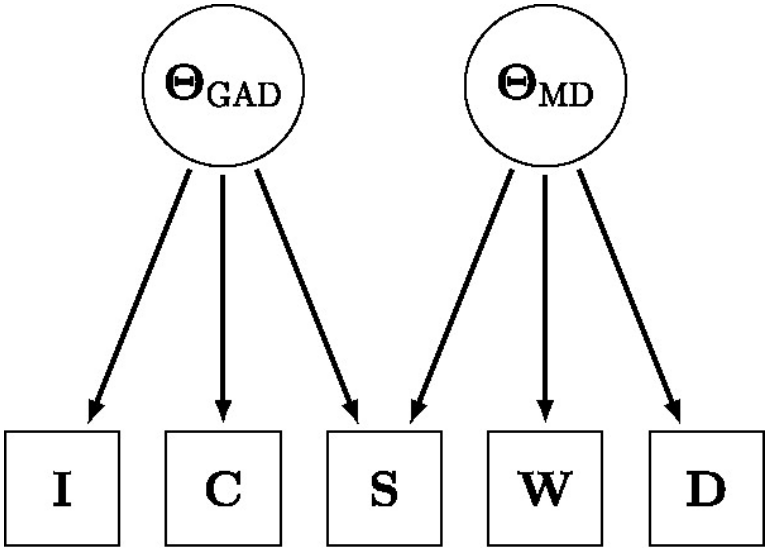
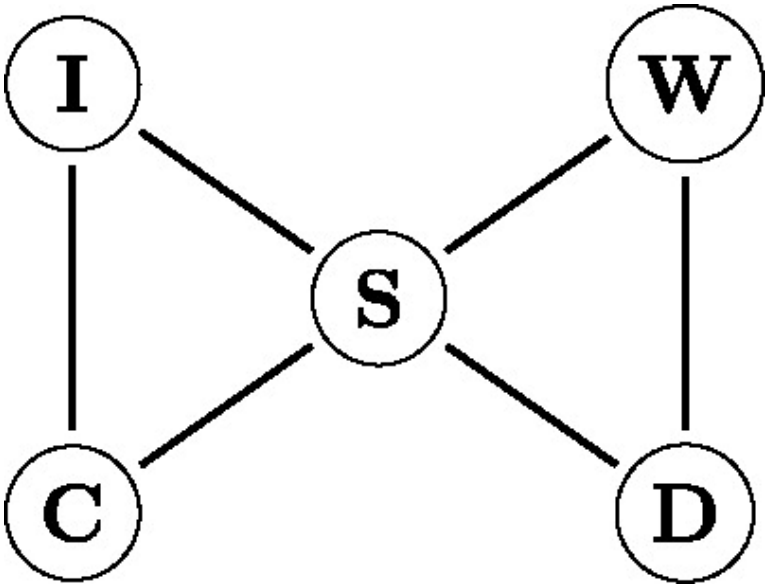
$$\Pr(\boldsymbol{X} = \boldsymbol{x}) = \int_{-\infty}^{\infty} f(\boldsymbol{\theta}) \Pr(\boldsymbol{X} = \boldsymbol{x} \mid \boldsymbol{\Theta} = \boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{\theta}$$

With:

$$\Pr(X_i = x_i \mid \boldsymbol{\Theta} = \boldsymbol{\theta}) = \frac{\exp\left(x_i \left(\boldsymbol{\alpha}_i^\top \boldsymbol{\theta} - \delta_i\right)\right)}{\sum_{x_i} \exp\left(x_i \left(\boldsymbol{\alpha}_i^\top \boldsymbol{\theta} - \delta_i\right)\right)}$$

$$\boldsymbol{\Theta} \mid \boldsymbol{X} = \boldsymbol{x} \sim N\left(\pm\frac{1}{2}\boldsymbol{A}^\top \boldsymbol{x}, \sqrt{\frac{1}{2}}\boldsymbol{I}\right)$$





I: irritability, C: chronic worrying, W: weight problems, D: depressed mood, S: sleep problems. Example from Marsman, M., Borsboom, D., Kruis, J., Epskamp, S., van Bork, R., Waldorp, L. J., van der Maas, H. L. J. & Maris, G. K. J. (2018). An introduction to Network Psychometrics: Relating Ising network models to item response theory models. *Multivariate Behavioral Research, 53(1)*, 15-35.

The Gaussian Graphical Model

Now assume $X_i \in \mathbb{R}\,\forall_i$ and the log of all potential functions are *linear*, and include variance potential function $\phi_{ii}(x_i)$:

$$\ln \phi_i(x_i) = \tau_i x_i$$
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We obtain:

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Now let $\boldsymbol{\Sigma} = \boldsymbol{K}^{-1}$ and $\boldsymbol{\mu} = \boldsymbol{\Sigma}\boldsymbol{\tau}$:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \exp\left(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{x} - \boldsymbol{\mu})\right)$$

Which implies $\boldsymbol{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$!

The Gaussian Graphical Model (GGM)

The precision matrix $\boldsymbol{K} = \boldsymbol{\Sigma}^{-1}$ encodes a network structure! Furthermore, this matrix can be standardized to *partial correlation coefficients*:

$$\mathrm{Cor}\left(X_i, X_j \mid \boldsymbol{X}_{-(i,j)}\right) = -\frac{\kappa_{ij}}{\sqrt{\kappa_{ii}}\sqrt{\kappa_{jj}}} = \omega_{ij}$$

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Combining this information, we can form a psychometric model:

$$\boldsymbol{\Sigma} = \boldsymbol{\Delta}\left(\boldsymbol{I} - \boldsymbol{\Omega}\right)^{-1}\boldsymbol{\Delta}$$

- $\boldsymbol{\Delta}$ is a diagonal scaling matrix
- $\boldsymbol{\Omega}$ is a symmetrical matrix with 0 on the diagonal and *partial correlation coefficients* on offdiagonal elements

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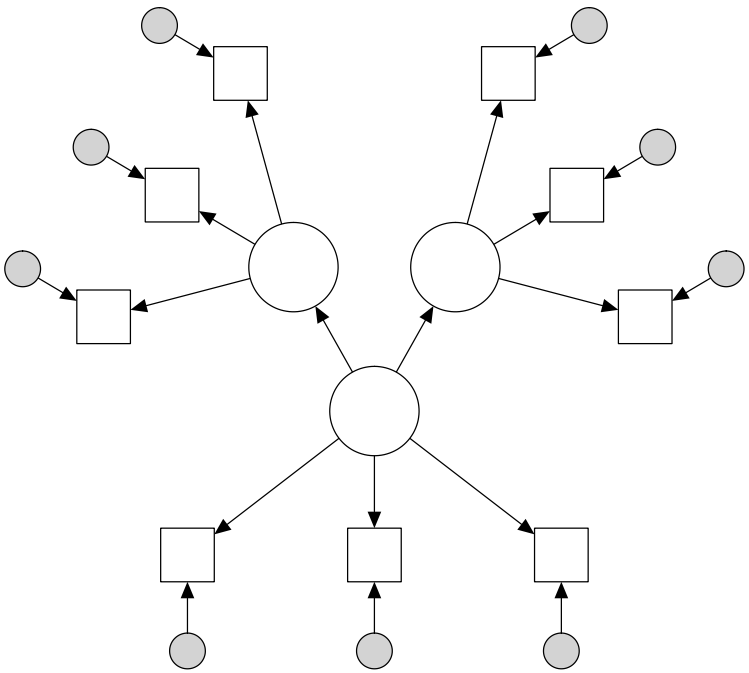
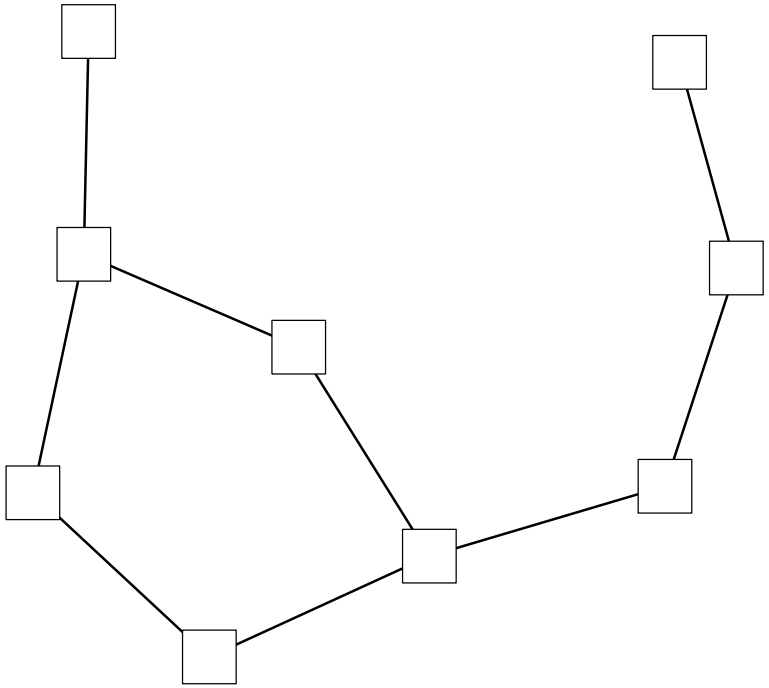
- Bayesian estimation methods are also promising
  - See the symposium: Network Psychometrics 1: Advances in Network Structure Estimation, Thursday 9:45-11:15

SEM and GGM

Structural Equation Modeling

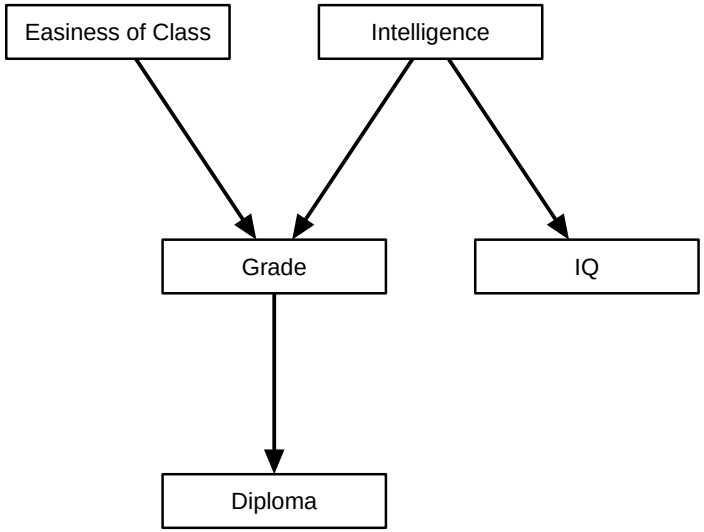


Gaussian Graphical Model

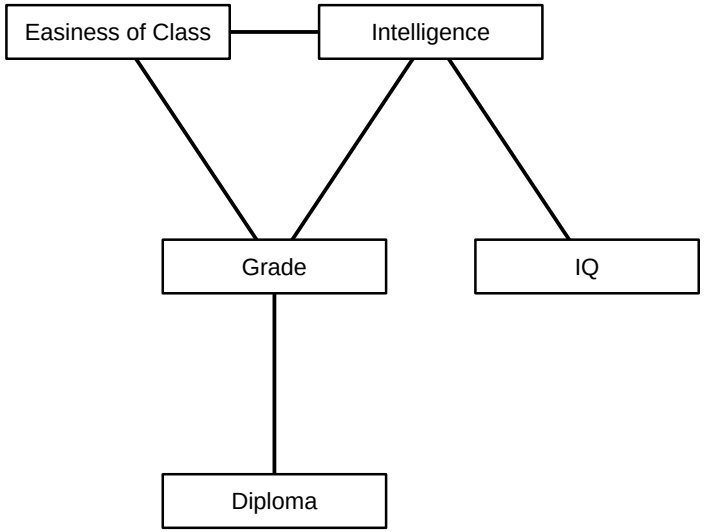


- Both models imply a variance-covariance matrix $\boldsymbol{\Sigma}$, aimed to closely resemble the sample variance-covariance matrix $\boldsymbol{S}$ with positive degrees of freedom.

Causal model



Markov Random Field



- MRF is uniquely identified and well parameterized
- MRF allows for exploratory hypothesis-generating insight in possible causal structures

Implementing networks in SEM

The variance-covariance matrices in a SEM model can be modeled as a network.

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Latent networks:

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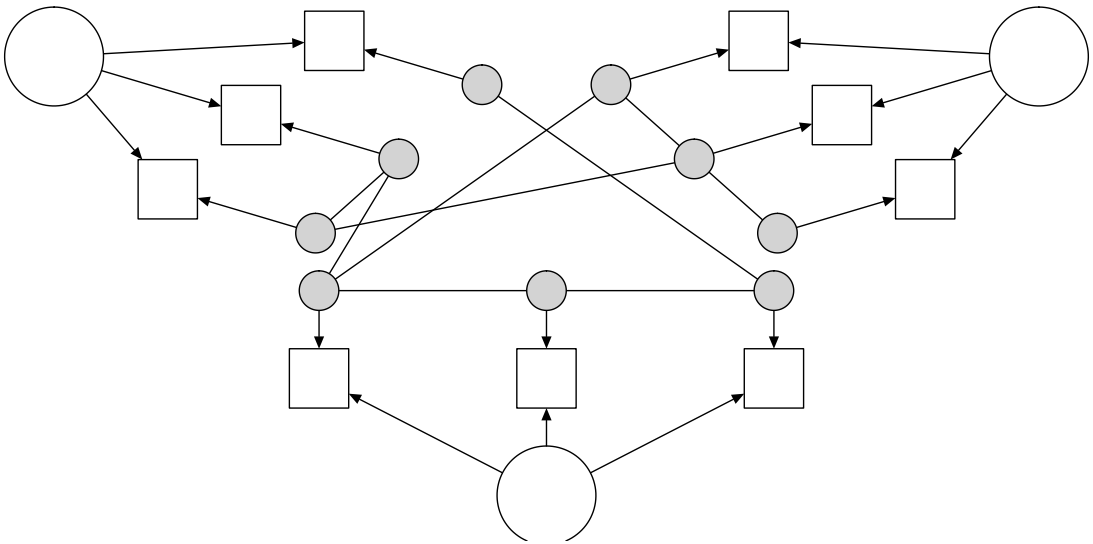
Residual Network Modeling (RNM)

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Psi}\boldsymbol{\Lambda}^\top + \boldsymbol{\Delta}_{\boldsymbol{\Theta}} \left(\boldsymbol{I} - \boldsymbol{\Omega}_{\boldsymbol{\Theta}}\right)^{-1} \boldsymbol{\Delta}_{\boldsymbol{\Theta}}$$

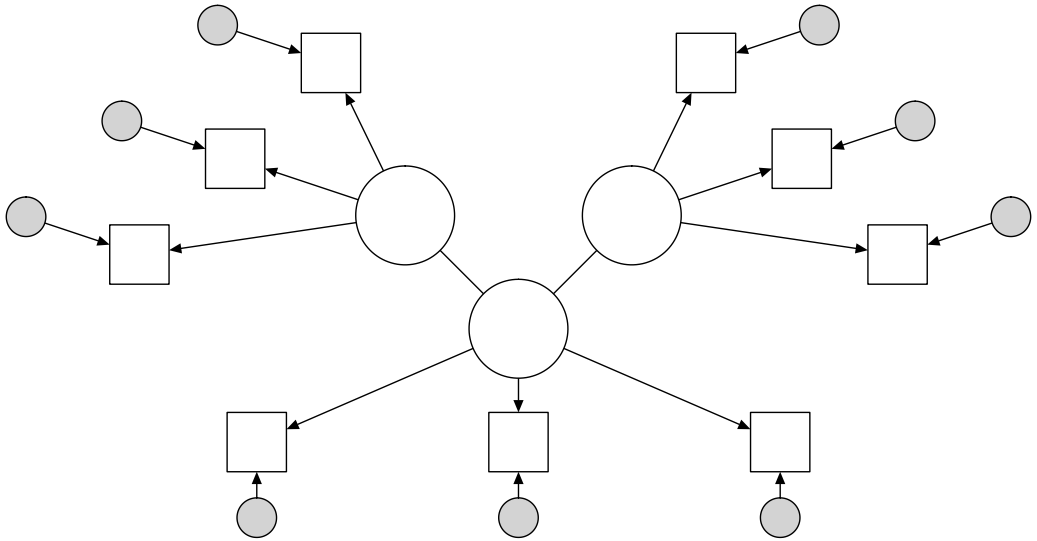
- Network is formed at the residuals of SEM
- Model a network while not assuming no unobserved common causes
- Model a latent variable structure without the assumption of local independence



Latent Network Modeling (LNM)

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Delta}_{\boldsymbol{\psi}} \left(\boldsymbol{I} - \boldsymbol{\Omega}_{\boldsymbol{\psi}}\right)^{-1} \boldsymbol{\Delta}_{\boldsymbol{\psi}}\boldsymbol{\Lambda}^{\top} + \boldsymbol{\Theta}$$

- Models conditional independence relations between latent variables as a network
- Model networks between latent variables
- Exploratory search for conditional independence relationships between latents

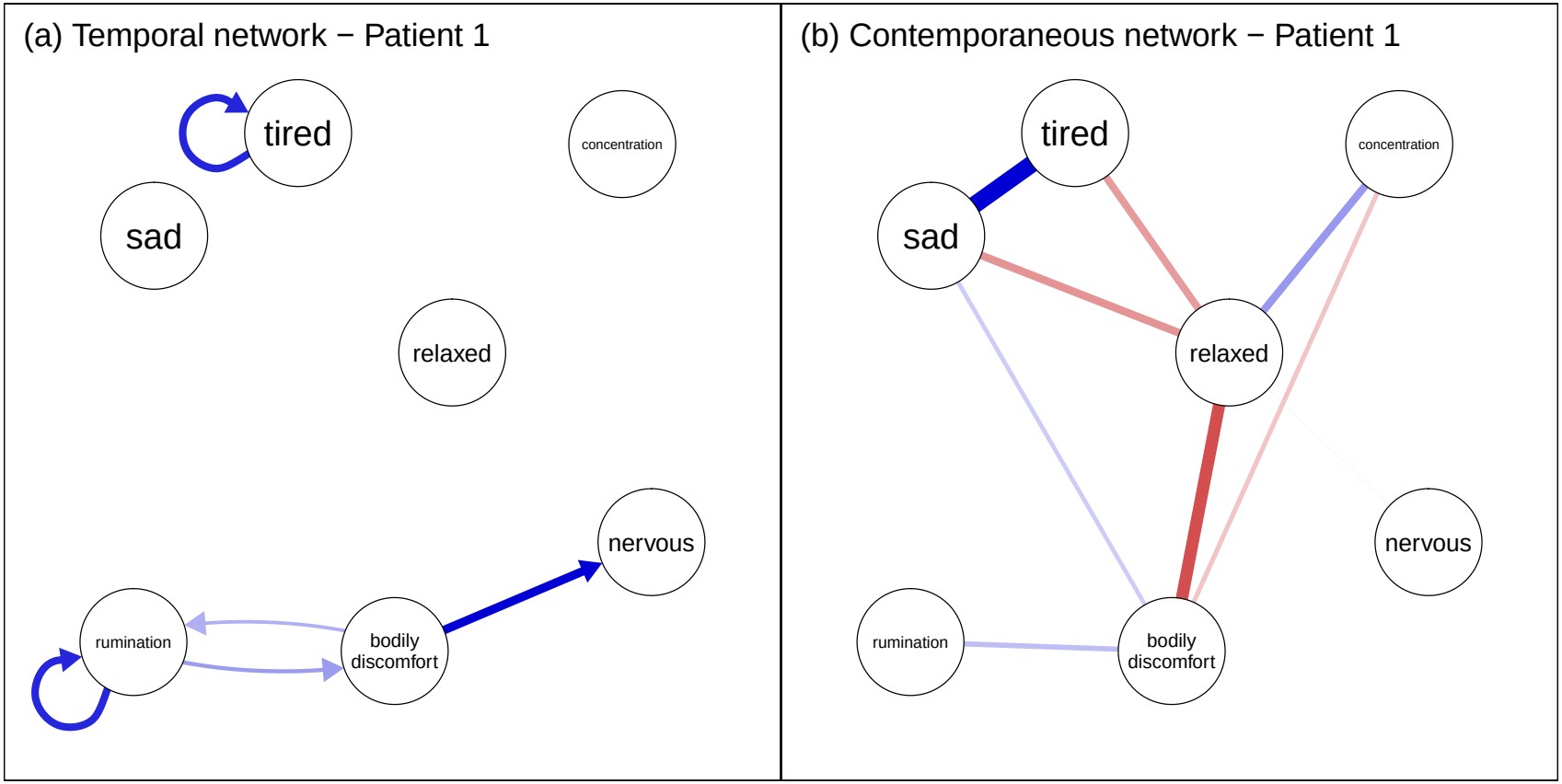


Time-series Analysis

Graphical Vector Auto-regression (VAR)

$$\boldsymbol{X}_t \mid \boldsymbol{x}_{t-1} \sim N\left(\boldsymbol{B}\boldsymbol{x}_{t-1}, \boldsymbol{\Theta}\right)$$

- Variables assumed centered
- $\boldsymbol{B}$ encodes the *temporal network*
  - Temporal prediction
- $\boldsymbol{\Theta}^{-1}$ encodes the *contemporaneous network*
  - GGM
- Graphical VAR model
  - Wild, B., Eichler, M., Friederich, H. C., Hartmann, M., Zipfel, S., & Herzog, W. (2010). A graphical vector autoregressive modelling approach to the analysis of electronic diary data. BMC medical research methodology, 10(1), 28.



- Contemporaneous network: conditional concentration given $t-1$
- Temporal network: regression coefficients between $t-1$ and t

Estimation

- Multivariate multiple regression on lagged variables
- Edge selection via significance thresholding, model selection (LNM) or LASSO regularization

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Important assumptions

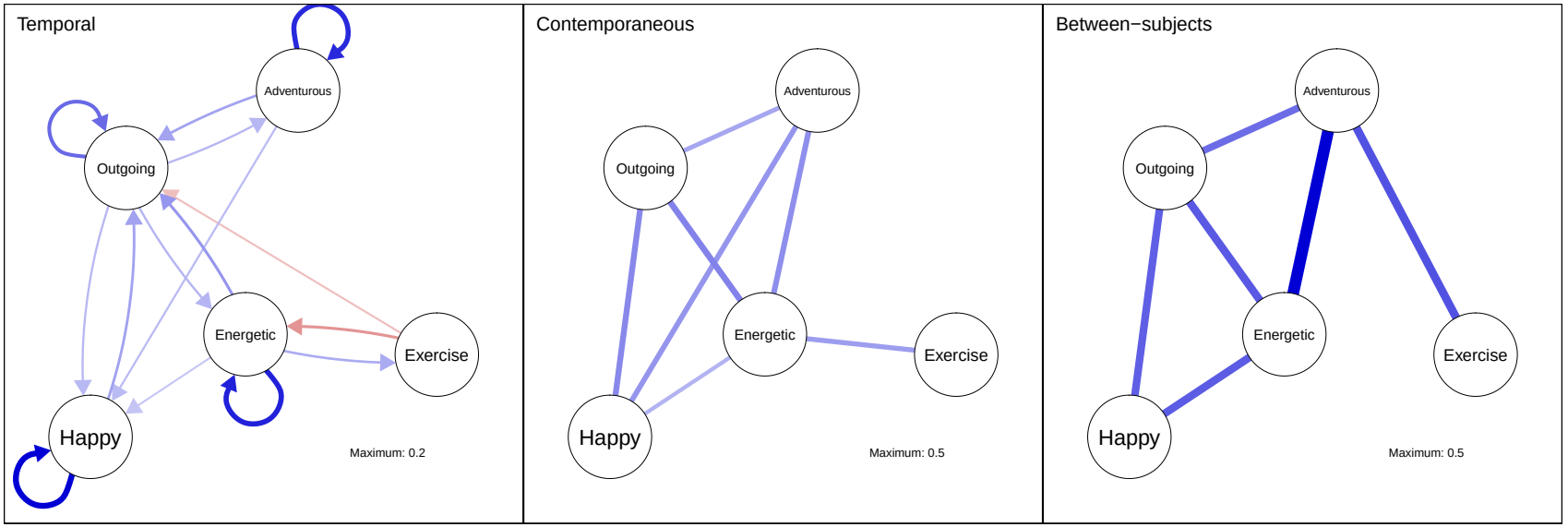
- Stationarity
  - Means
  - Network structure
- Equidistant measurements

Multi-level VAR

Individual network model per person:

$$\boldsymbol{X}_t^{(p)} \mid \boldsymbol{x}_{t-1}^{(p)} = N\left(\boldsymbol{\mu}^{(p)} + \boldsymbol{B}^{(p)}\left(\boldsymbol{x}_{t-1}^{(p)} - \boldsymbol{\mu}^{(p)}\right), \boldsymbol{\Theta}^{(p)}\right)$$

- Temporal and Contemporaneous network per person
- Fixed effects temporal and contemporaneous effects
- Correlations of the means can be used to construct a between-subjects network
  - Partial correlation network between means


Temporal
Adventurous
Outgoing
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Exercise
Happy
Maximum: 0.2
Contemporaneous
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Between−subjects
Adventurous
Outgoing
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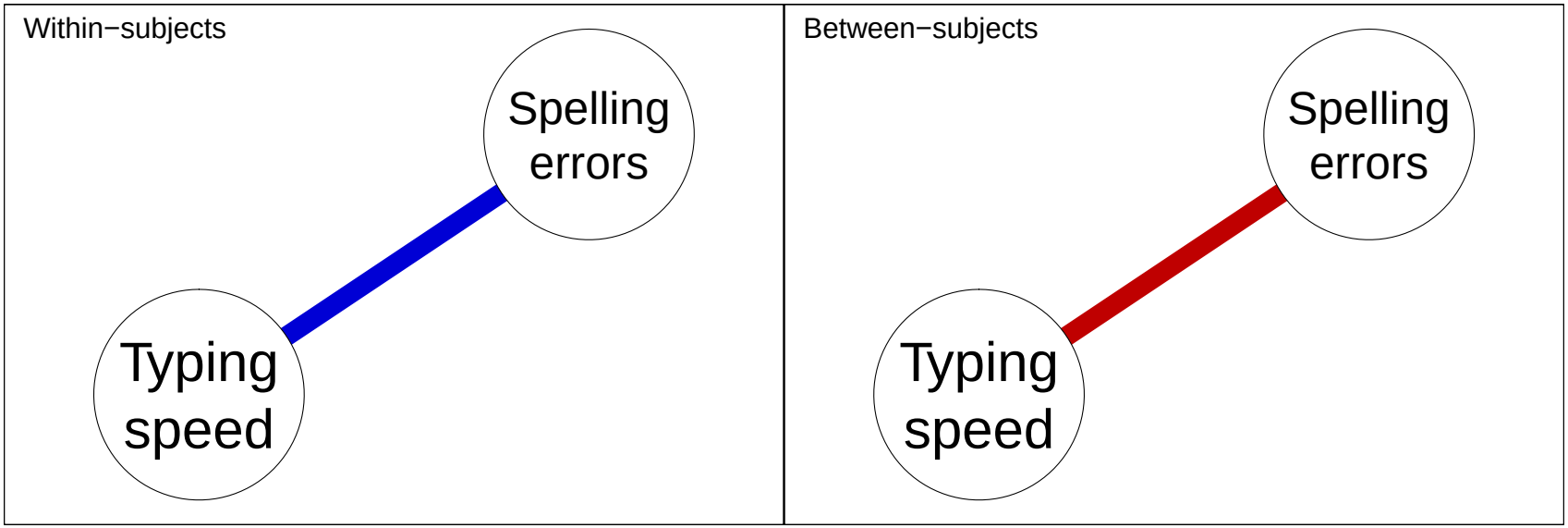
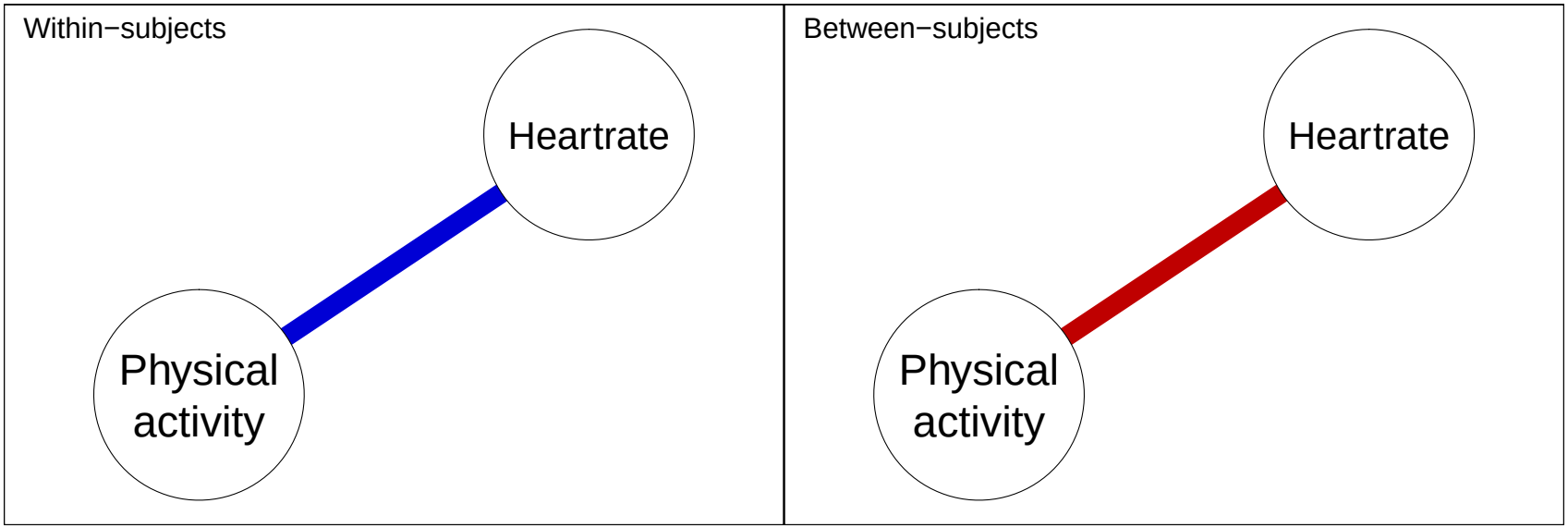
Estimation

Three Methods of Estimating GVAR Models With $n > 1$ Subjects

	Pooled and individual LASSO estimation	Bayesian multilevel	Two-step frequentist multilevel
Software	*graphicalVAR* (Epskamp, 2017b); *sparseTSCGM* (Abegaz & Wit, 2015).	*MPlus* 8 (Muthén & Muthén, 2017; Asparouhov et al., 2016); *mlVAR* (wrapper around Mplus).	*mlVAR* (Epskamp, Deserno, & Bringmann, 2017).
Estimation	(1) Joint multivariate LASSO estimation with EBIC model selection (Abegaz & Wit, 2013) of within-subjects centered data to obtain fixed effects temporal and contemporaneous networks. (2) glasso algorithm with EBIC model selection (Foygel & Drton, 2010) on sample means of subjects to obtain between-subject network. (3) Step (1) repeated for each individual dataset to obtain subject-specific networks.	MCMC sampling from multivariate hierarchical model (e.g., Schuurman, Grasman, & Hamaker, 2016).	(1) Sequential univariate multilevel regression models on previous measurement (similar to Bringmann et al., 2013), with within-subject centered lagged variables as within-subjects level predictors and sample-means of all other variables as between-subjects predictor. (2) Sequential multilevel regression models using the residuals of (1): residuals of one variable are predicted by residuals of all other variables in the same measurement occasion.



Example based on Hamaker, E. L. (2012). Why Researchers Should Think 'Within-Person': A Paradigmatic Rationale. *Handbook of Research Methods for Studying Daily Life*. The Guilford Press New York, NY, 43–61.



Example provided by Ellen Hamaker and based on Hoffman, L. (2015). *Longitudinal analysis: Modeling within-person fluctuation and change*. New York, NY, USA: Routledge.

Future Directions & Conclusion

Current state of Network Psychometrics

- Applied in over 100 empirical papers

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Title	Speaker / Chair	Day	Time
Symposium 1: Cognitive Development: Network Psychometric Approaches	Maarten Marsman	Tuesday	11:00 - 12:00
Symposium 3: Psychometric Developments in JASP	Don van den Bergh	Tuesday	13:30 - 15:00
Symposium 5: Modeling Intensive Longitudinal Data: Perks and Pitfalls	Laura Bringmann	Tuesday	15:20 - 16:50
Diagnosing Diagnostic Models: From von Neumanns Elephant to Model Equivalencies and Network Psychometrics	Matthias von Davier	Tuesday	17:00 - 17:50
Network Psychometrics: Current State and Future Directions	Sacha Epskamp	Tuesday	17:50 - 18:30
Symposium 8: Non-Cognitive Psychometric Theory and Assessment	Gunter Maris	Wednesday	08:30 - 10:00
Equivalent Dynamic Models	Peter Molenaar	Wednesday	10:15 - 11:00
Symposium 9: Timely Perspectives on Dynamic Models for Time Series and Panels	Eva Ceulemans; Janne Adolf	Wednesday	15:00 - 16:30
Network Analysis	Han van der Maas	Wednesday	16:30 - 18:00
Symposium 13: Network Psychometrics 1: Advances in Network Structure Estimation	Denny Borsboom	Thursday	09:45 - 11:15
Symposium 17: Network Psychometrics II: Psychometric Extensions to Network Modeling	Sacha Epskamp	Friday	08:30 - 10:00

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  - MRFs for ordinal data
  - Missing data handling
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Thank you for your attention!

- Website: `sachaepskamp.com`
  - Slides at: `sachaepskamp.com/presentations`
  - Dissertation at: `sachaepskamp.com/dissertation`
- Twitter: `twitter.com/SachaEpskamp`
- Join our Facebook group!
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