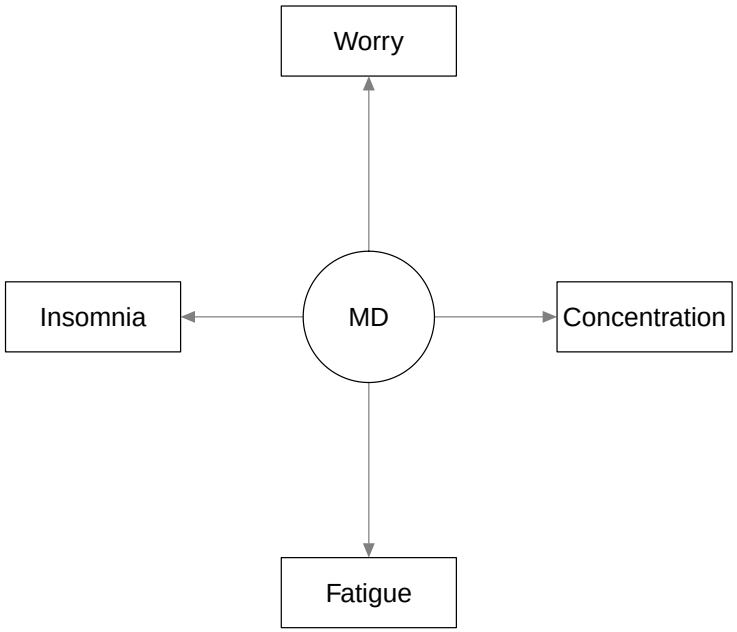
Network modeling of psychological processes

From exploration to theory formation

Sacha Epskamp

EMPG 2018

The Network Perspective



- Cramer, A. O. J., Waldorp, L. J., van der Maas, H., & Borsboom, D. (2010). Comorbidity: A network perspective. *Behavioral and Brain Sciences, 33*, 137-193.
- Borsboom, D. (2017). A network theory of mental disorders. *World Psychiatry, 16*, 513.

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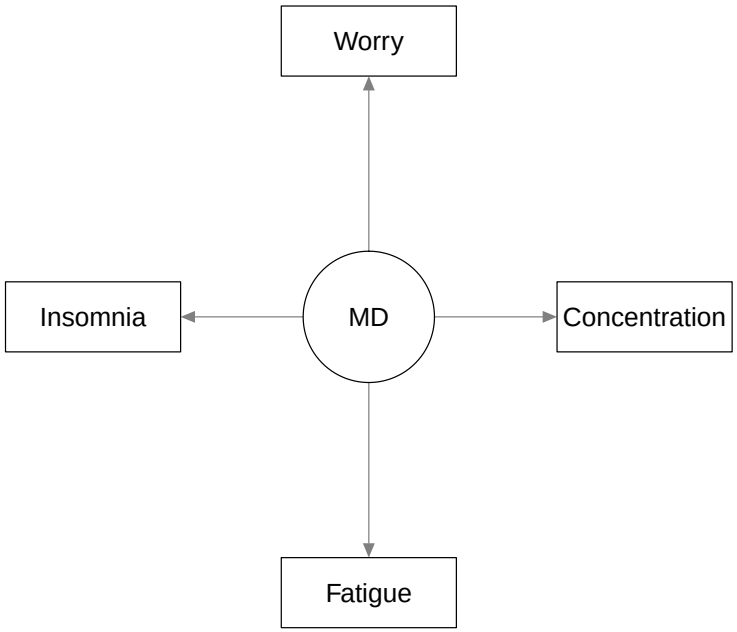

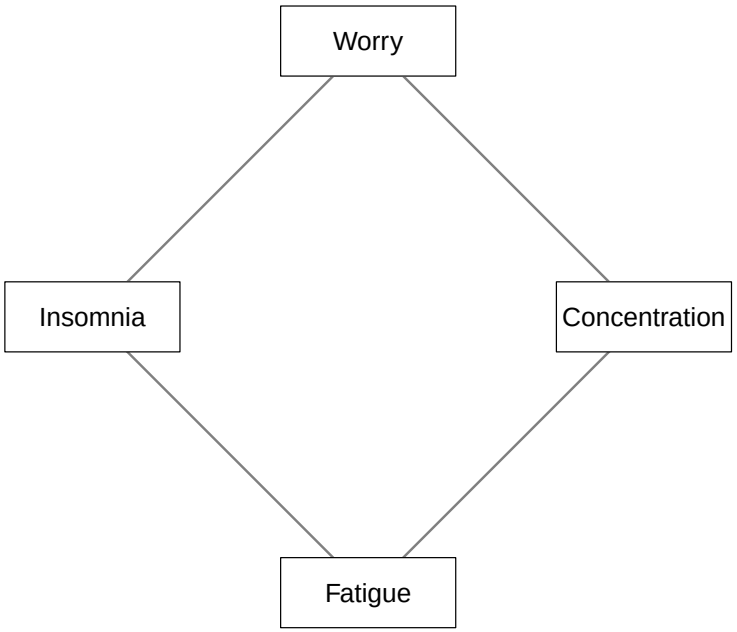

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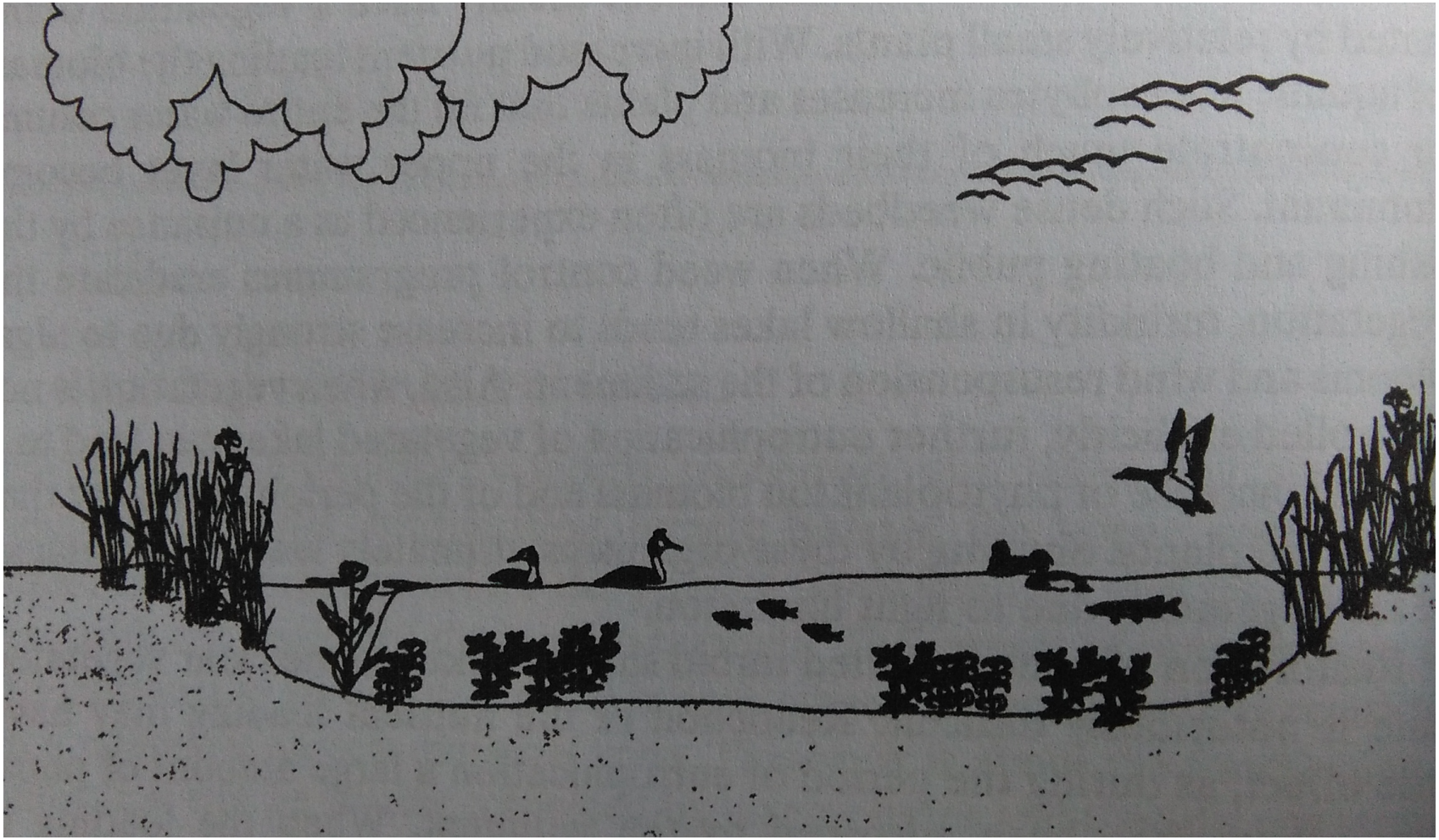
Analogy 1: Ecology



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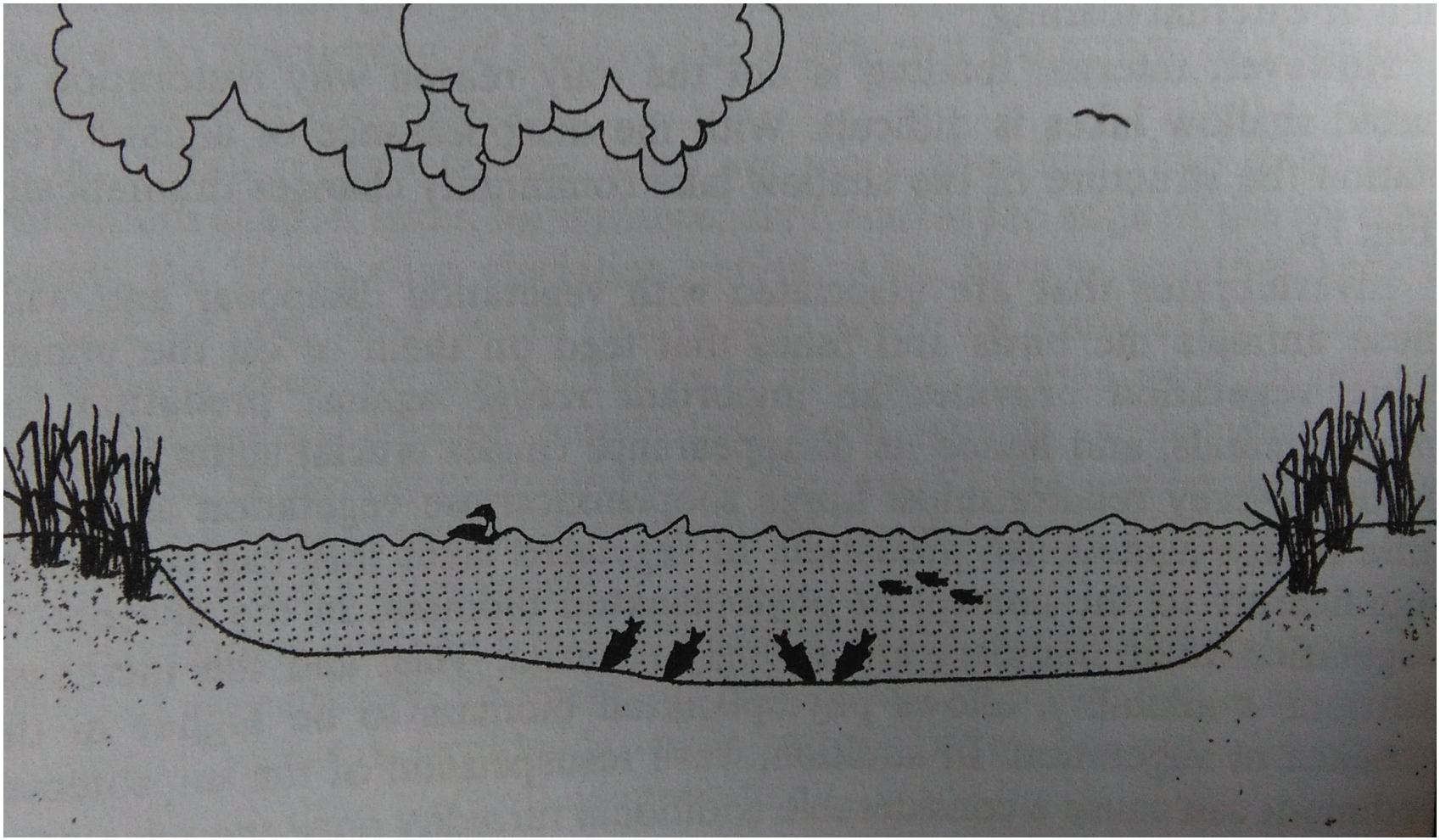

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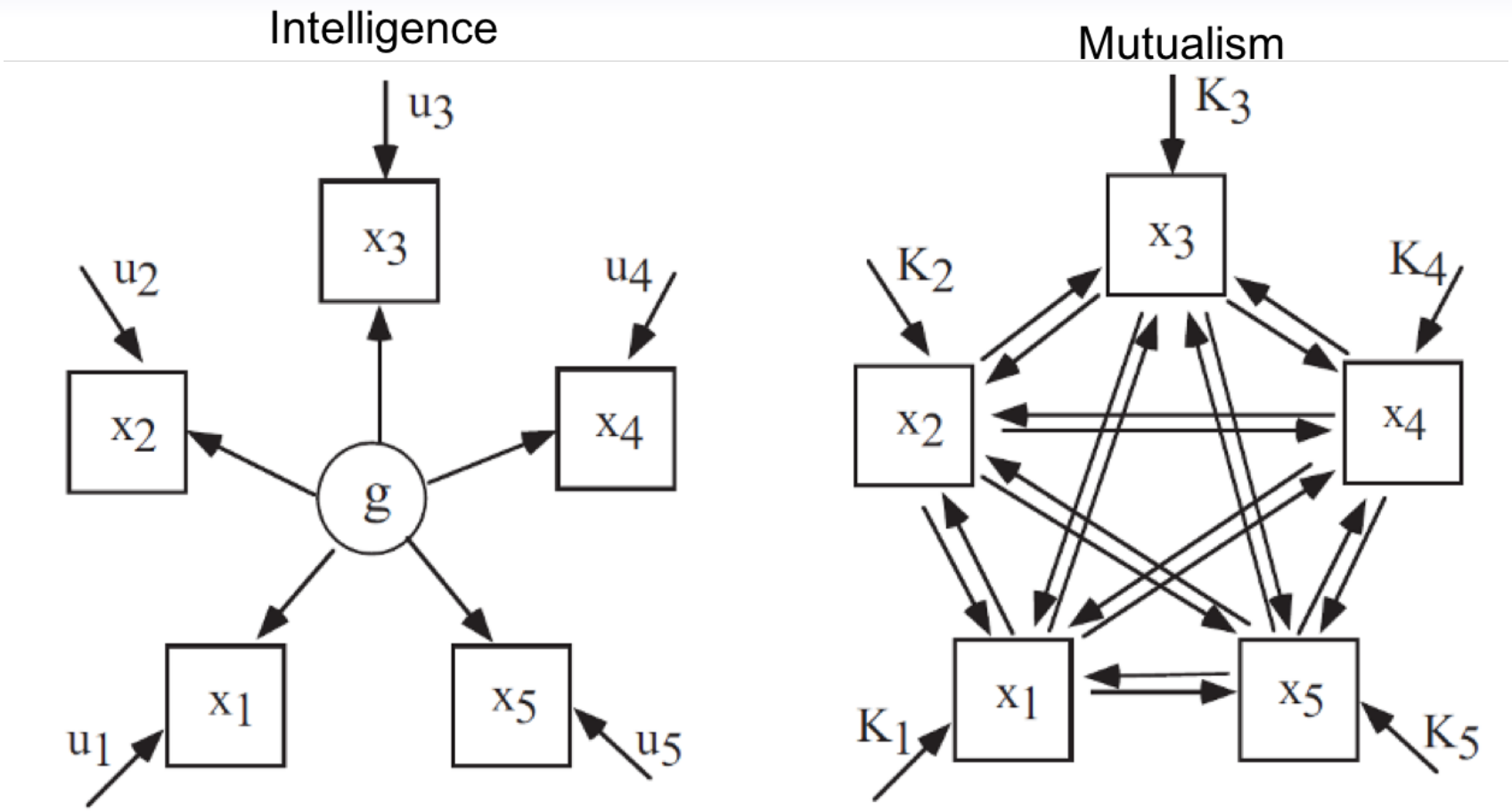
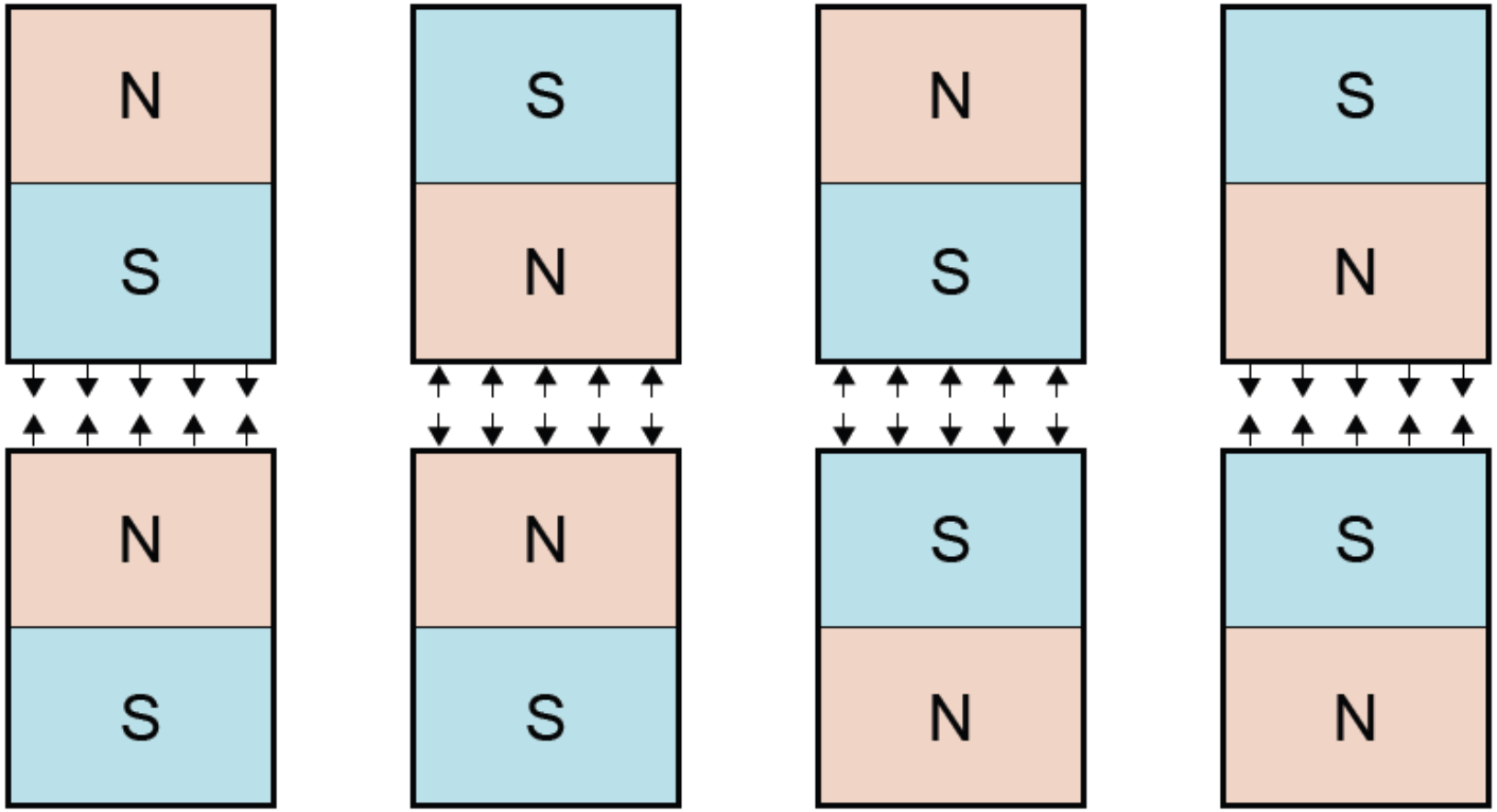
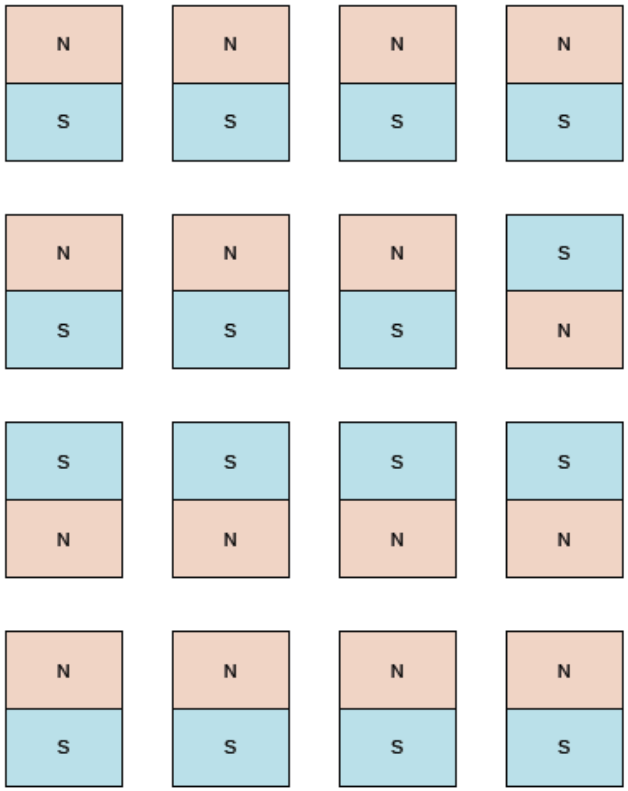


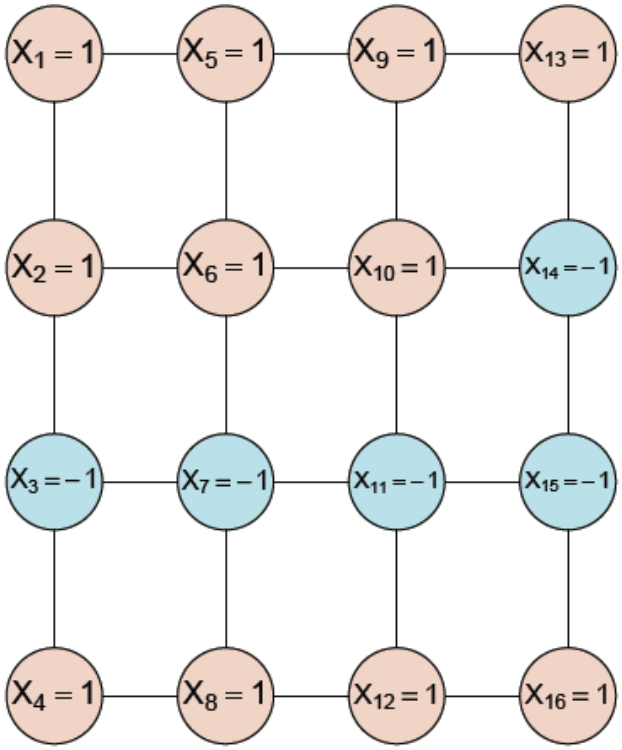
van der Maas, H. L., Dolan, C. V., Grasman, R. P., Wicherts, J. M., Huizenga, H. M., & Raijmakers, M. E. (2006). A dynamical model of general intelligence: The positive manifold of intelligence by mutualism. *Psychological Review, 113*(4), 842861. doi: 10.1037/ 0033-295X.113.4.842 .

Analogy 2: Ferromagnetism



The Ising model:

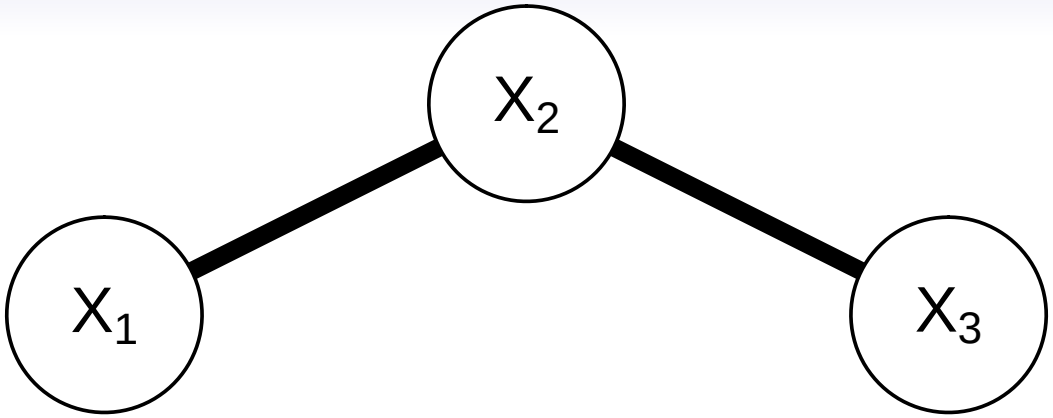


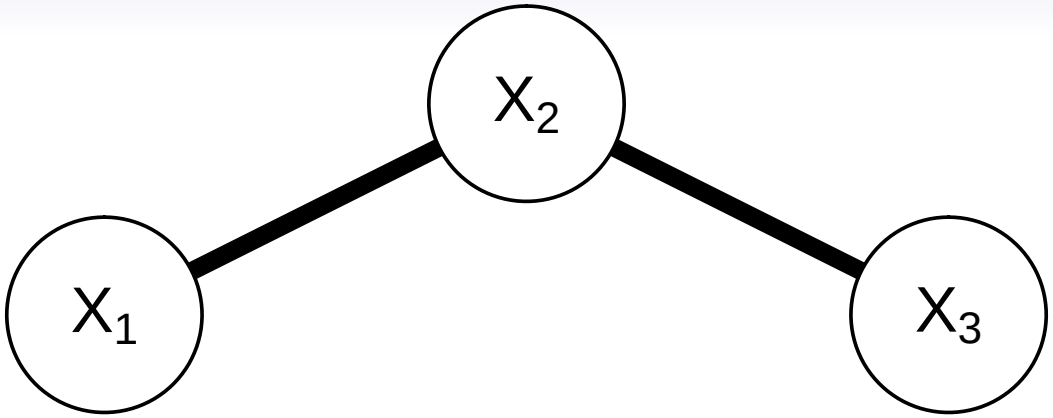


Analogy 3: Psychopathology as a Virus

Video...

Markov Random Fields

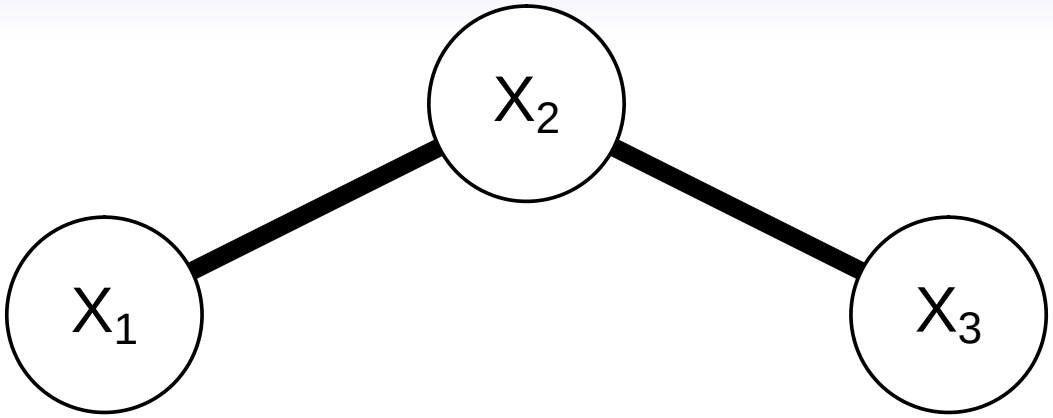

X_2
X_1
X_3



Graphs

$$G = \{V, E\}$$
$$V = \{1, 2, 3\}$$
$$E = \{(1, 2), (2, 3)\}$$

A graph is a set G consisting of two sets: V (set of nodes) and E (set of edges).



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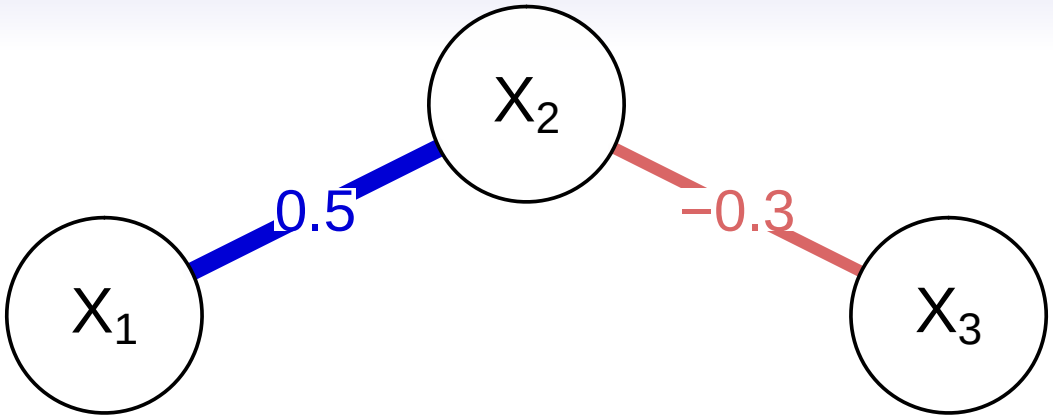
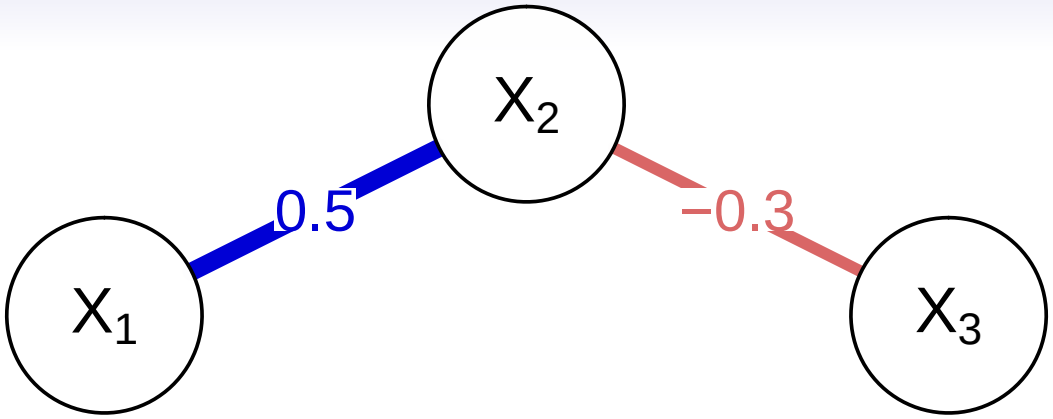
Markov Random Fields

$$X_i \perp\!\!\!\perp X_j \mid \boldsymbol{X}_{-(i,j)} = \boldsymbol{x}_{-(i,j)} \iff (i, j) \notin E$$

- Graphical representation: Two variables are connected if they are not conditionally independent
- Powerful characterization of joint likelihood between observed variables


X2
0.5
−0.3
X1
X3

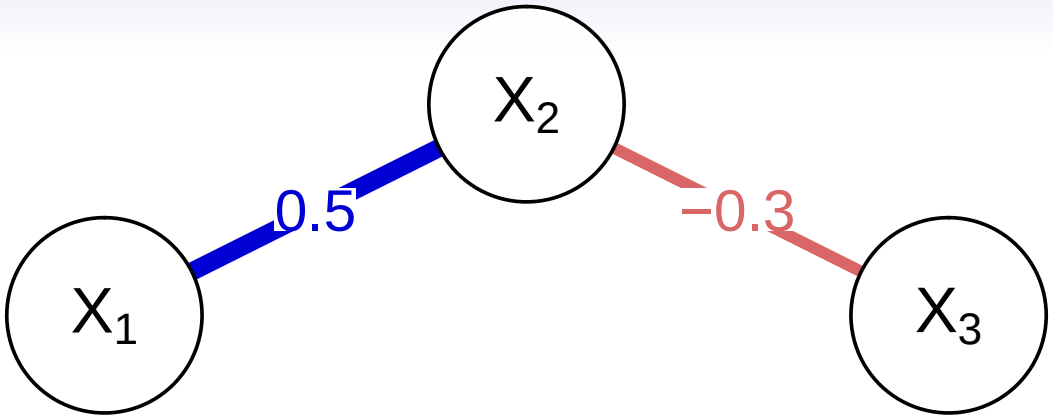


Weighted graphs

A weighted graph uses a *weights matrix* to characterize the strength of connection:

$$\mathbf{\Omega} = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & -0.3 \\ 0 & -0.3 & 0 \end{bmatrix}$$

Positive edges are drawn blue (or green) and negative edges red.



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Markov Random Fields

In MRFs, weights should indicate conditional independence:

$$X_i \perp\!\!\!\perp X_j \mid \boldsymbol{X}_{-(i,j)} = \boldsymbol{x}_{-(i,j)} \iff \omega_{ij} = 0$$

Furthermore, positive weights should be comparable to negative weights in strength of association

Markov Random Field

In a Markov random field (MRF) a joint probability distribution is formed by multiplying *potential functions* $\phi_C(\boldsymbol{x}_C) \geq 0$ for every *clique* in a graph (e.g., one variable, two connected varaibles, a set of three connected variables):

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \prod_{C \in \mathrm{cl}(G)} \phi_C(\boldsymbol{x}_C)$$

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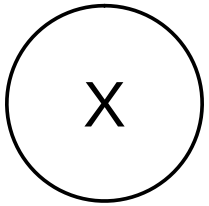
Pairwise MRFs only include potential function up to pairwise interactions:

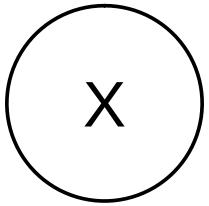
$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \prod_{i} \phi_i(x_i) \prod_{<ij>} \phi_{ij}(x_i, x_j)$$

These are typically used in network psychometrics



Suppose random variable X with realization x can take two outcomes: s_1 and s_2. We can make use of a potential function to map a unique weight to each outcome:

$$\phi(x) = \begin{cases} a & \text{if } x = s_1 \\ b & \text{if } x = s_2 \end{cases}$$



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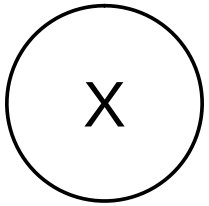
$$\Pr(X = x) \propto \phi(x)$$



Suppose random variable X with realization x can take two outcomes: s_1 and s_2. We can make use of a potential function to map a unique weight to each outcome:

$$\phi(x) = \begin{cases} a & \text{if } x = s_1 \\ 1/a & \text{if } x = s_2 \end{cases}$$

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We can add a different potential function for X_2:

$$\Pr(X_1 = x_1, X_2 = x_2) \propto \phi_1(x_1)\phi_2(x_2)$$

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Two parameters (one per potential function) used to model three probabilities (4 possible outcomes): not a saturated model.

We can add a potential function to model the *pairwise interaction* between X_1 and X_2:

$$\phi_{12}(x_1, x_2) = \begin{cases} c & \text{if } x_1 = s_1 \wedge x_2 = s_1 \\ d & \text{if } x_1 = s_2 \wedge x_2 = s_1 \\ e & \text{if } x_1 = s_1 \wedge x_2 = s_2 \\ f & \text{if } x_1 = s_2 \wedge x_2 = s_2 \end{cases}$$

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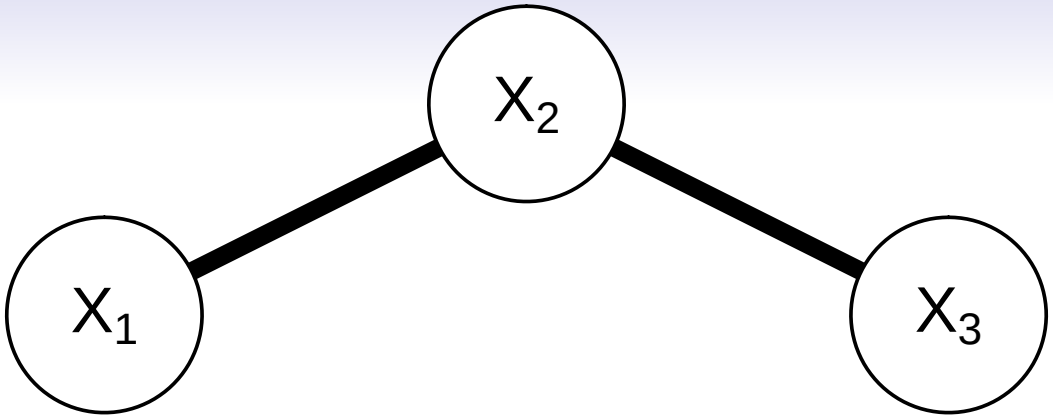
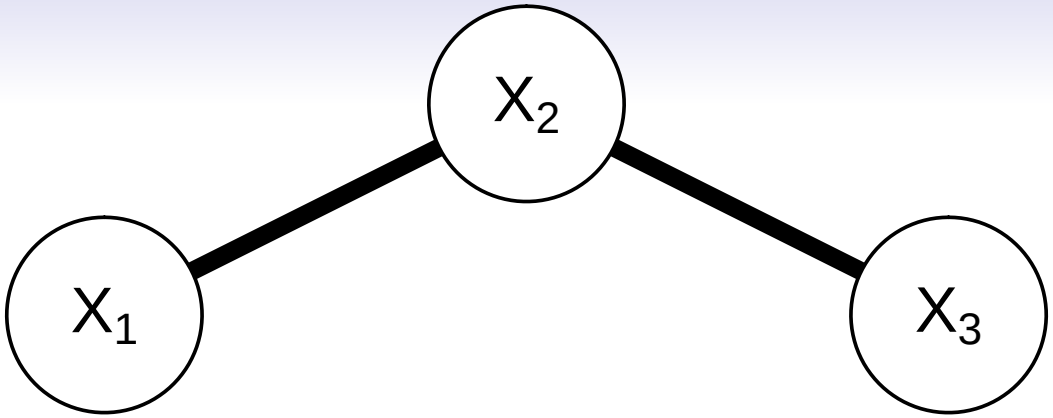


Let $\boldsymbol{X}^\top = \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix}$ with realization $\boldsymbol{x}$. No interaction between X_1 and X_3, $\phi_{13}(x_1, x_3) = 1 \, \forall_{x1,x3}$, gives:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_{12}(x_1, x_2)\phi_{23}(x_2, x_3)$$



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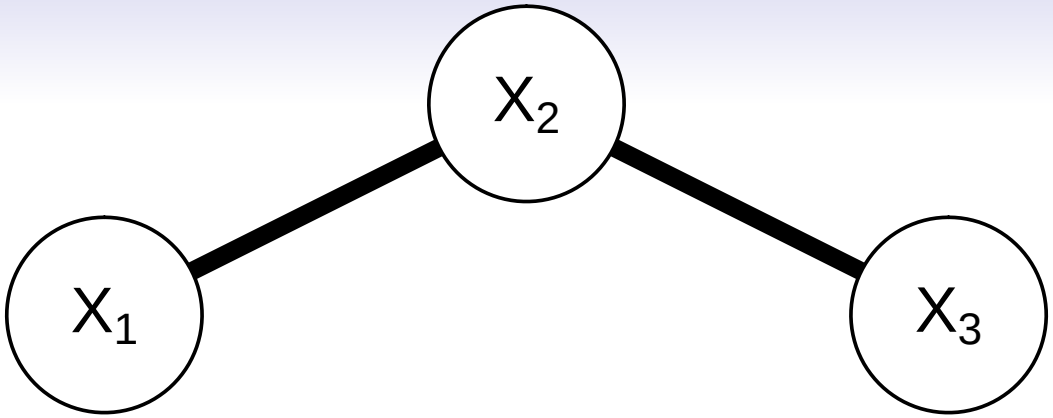
X_1 and X_3 are *conditionally* independent:

$$\begin{aligned} \Pr\left(\boldsymbol{X}_{-(2)} = \boldsymbol{x}_{-(2)} \mid X_2 = x_2\right) &\propto \phi_1(x_1)\phi_{12}(x_1, x_2)\phi_3(x_3)\phi_{23}(x_2, x_3) \\ &\propto \phi_1^*(x_1)\phi_3^*(x_3) \end{aligned}$$



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But not *marginally* independent:

$$\Pr\left(\boldsymbol{X}_{-(2)} = \boldsymbol{x}_{-(2)}\right) \propto \sum_{x_2} \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_{12}(x_1, x_2)\phi_{23}(x_2, x_3)$$

Ising Model

Pairwise Markov Random Field:

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Now encode $S_1 = 1$ and $S_2 = -1$, we can define without loss of information (in binary setting):

$$\ln \phi_i(x_i) = \tau_i x_i$$
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This gives the *Ising Model*:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \exp\left(\sum_i \tau_i x_i + \sum_{<ij>} \omega_{ij} x_i x_j\right)$$

in which τ_i encodes the potential of X_i to be in state 1 or -1 and ω_{ij} encodes the strength of connection between X_i and X_j.

Estimating an Ising model

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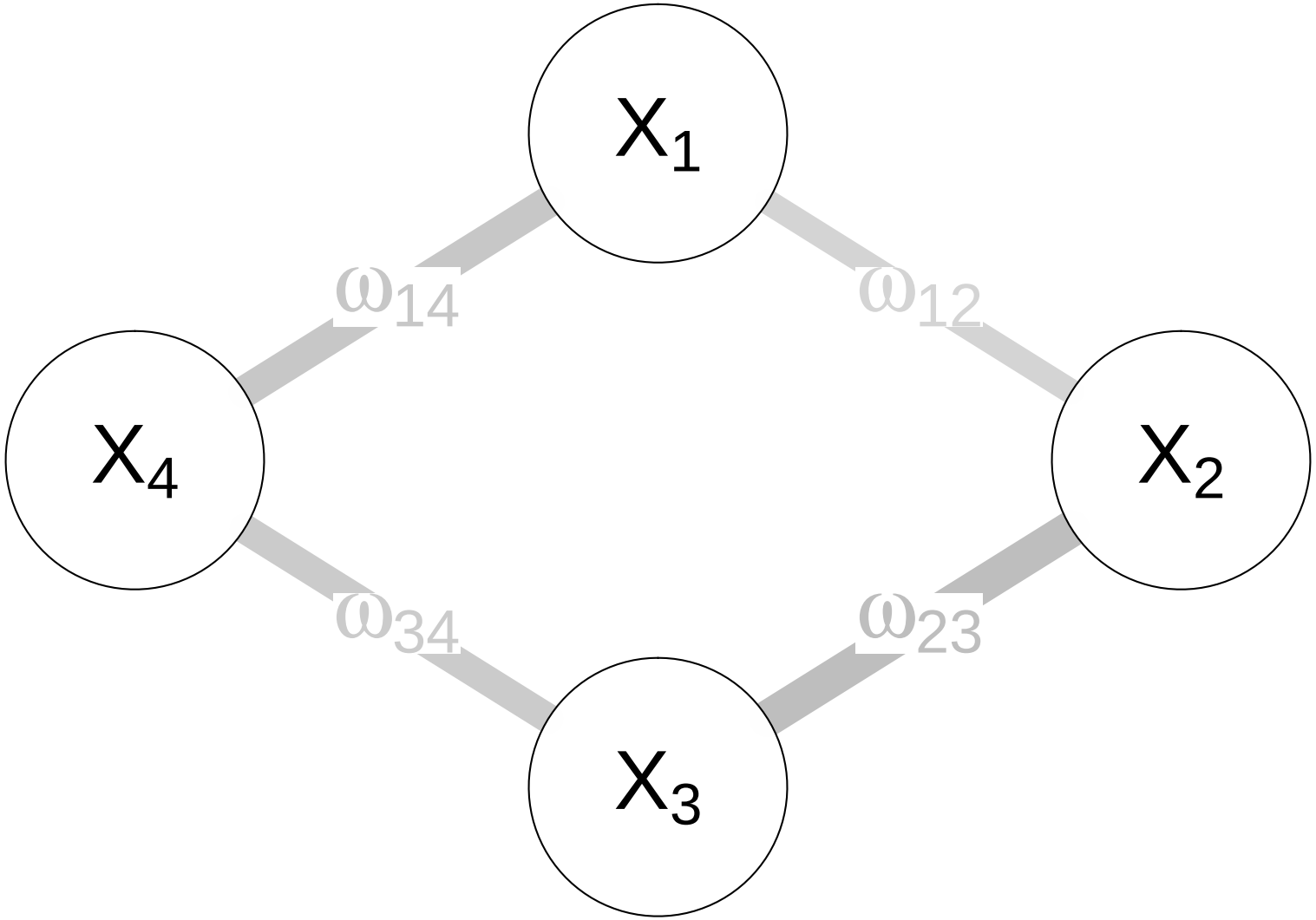
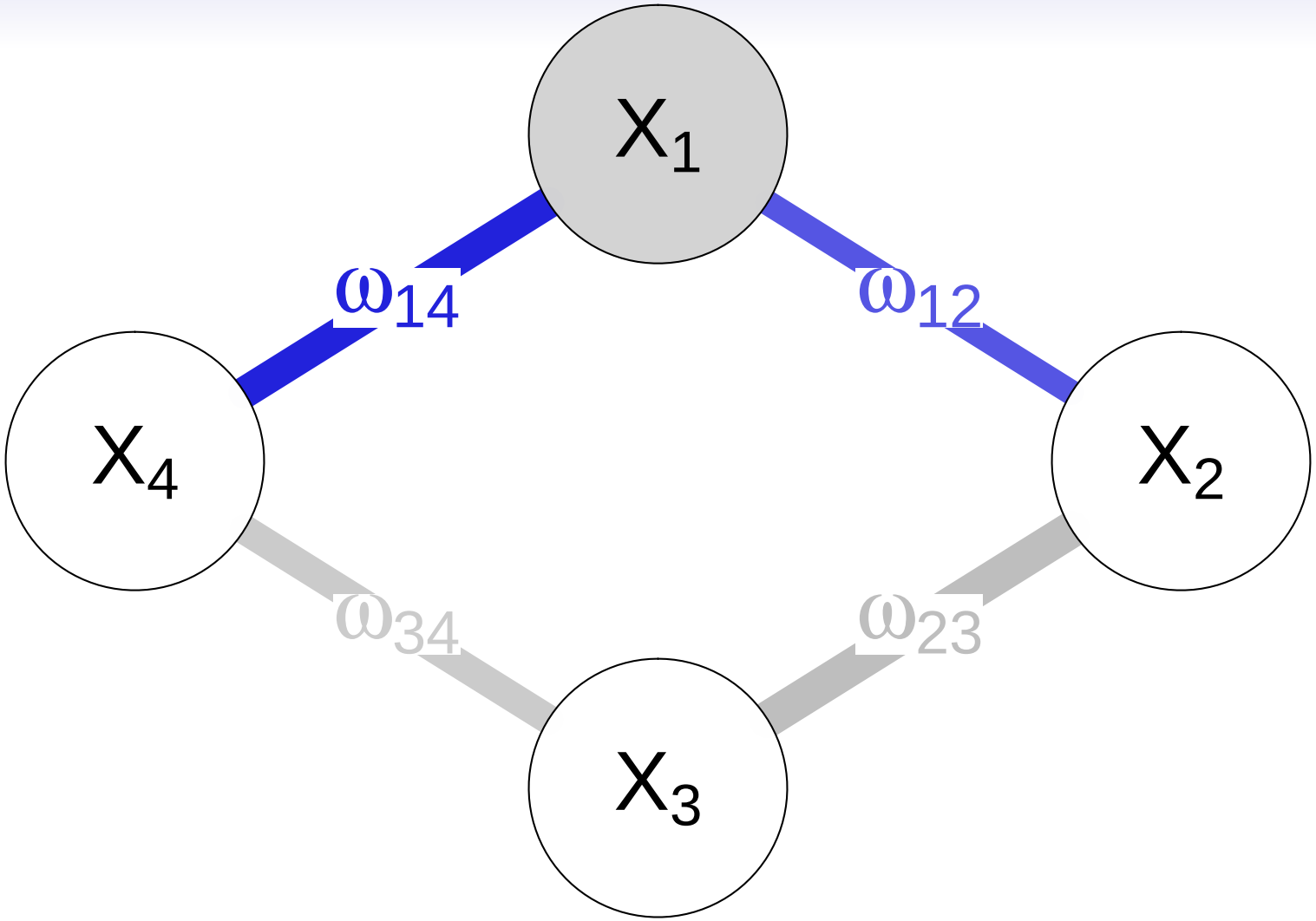
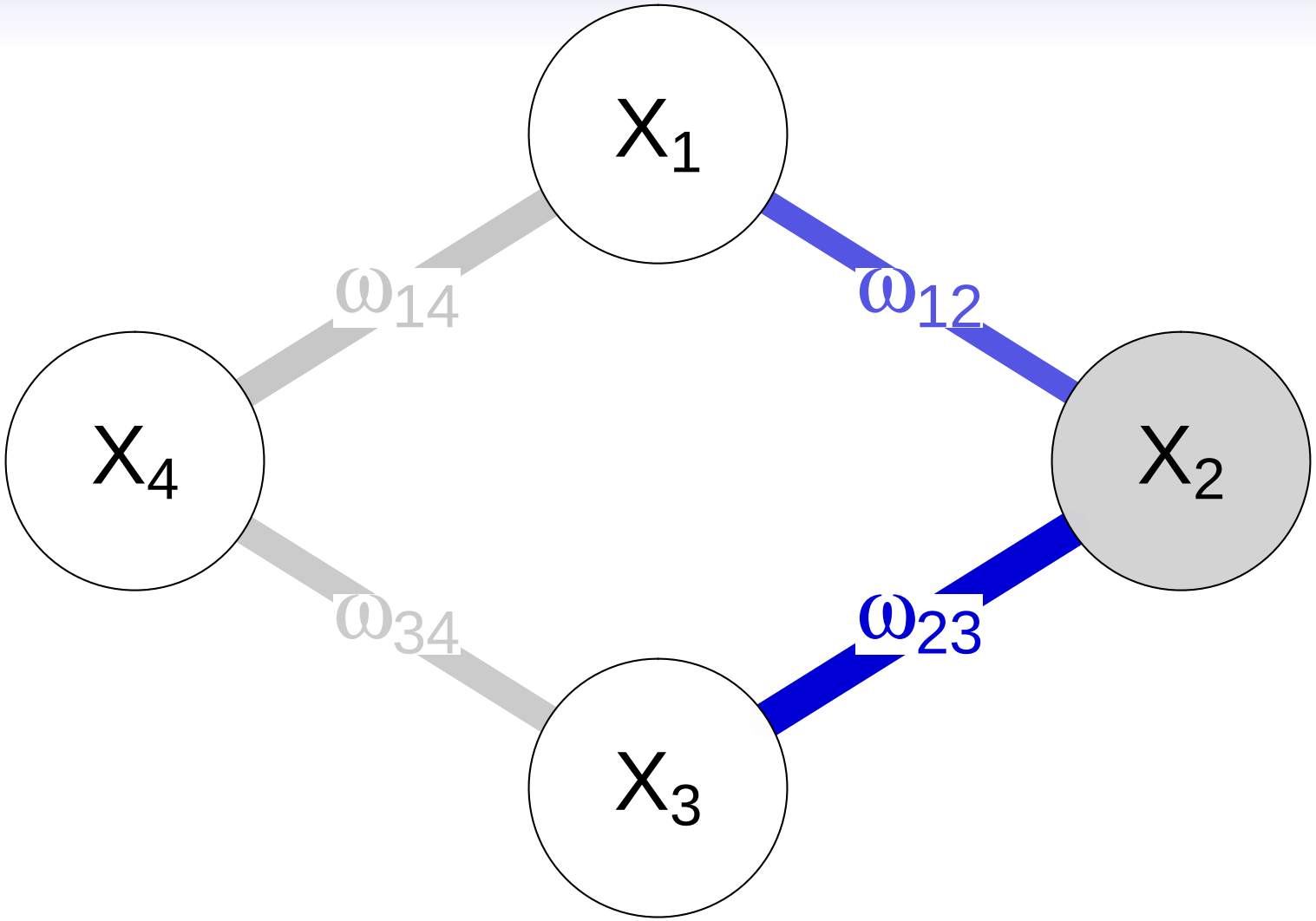
- This is a *multiple logistic regression* model!
- The Ising model can thus be estimating by performing several multiple logistic regressions

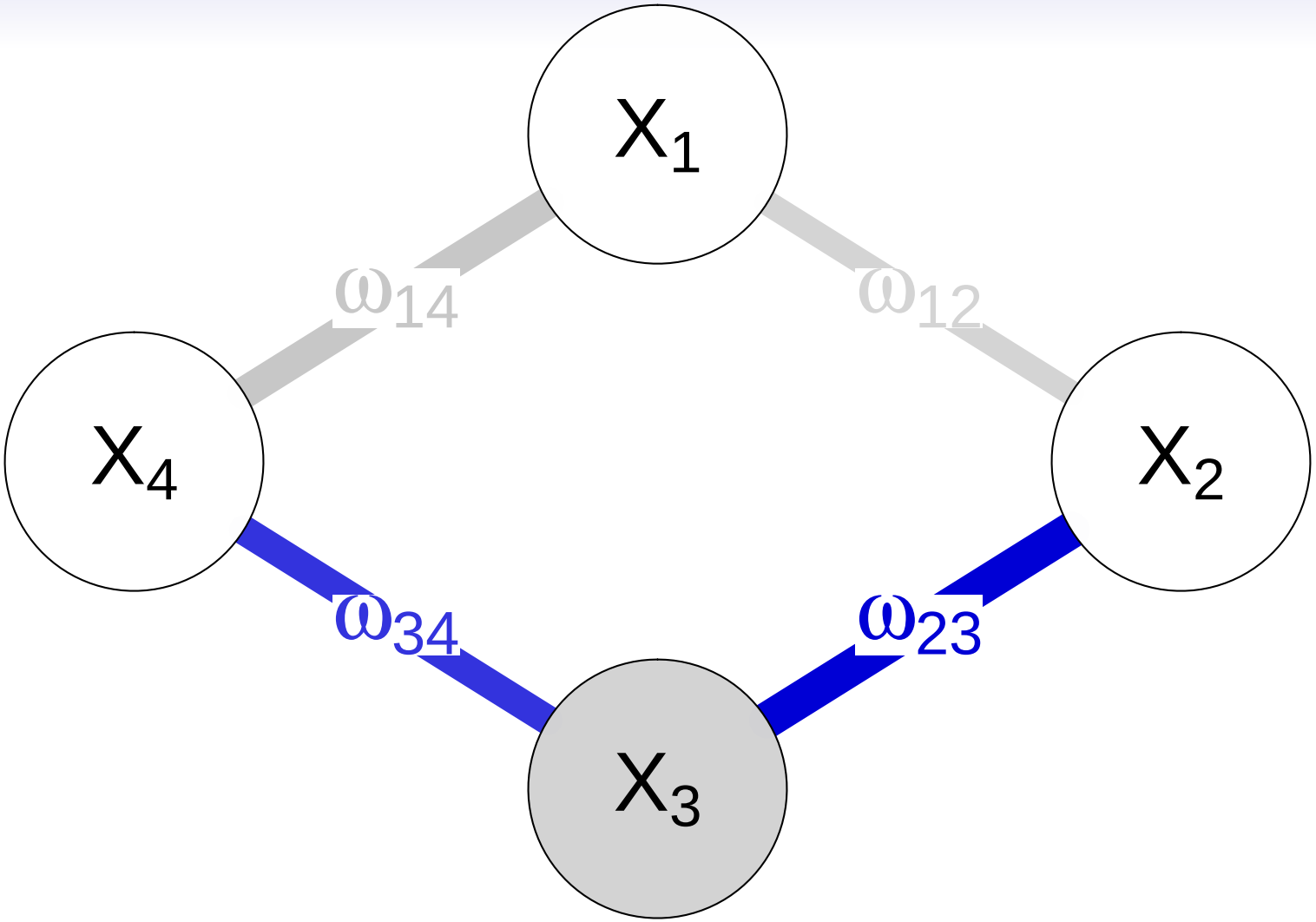

X_1
X_2
X_3
X_4
ω_{14}
ω_{12}
ω_{34}
ω_{23}



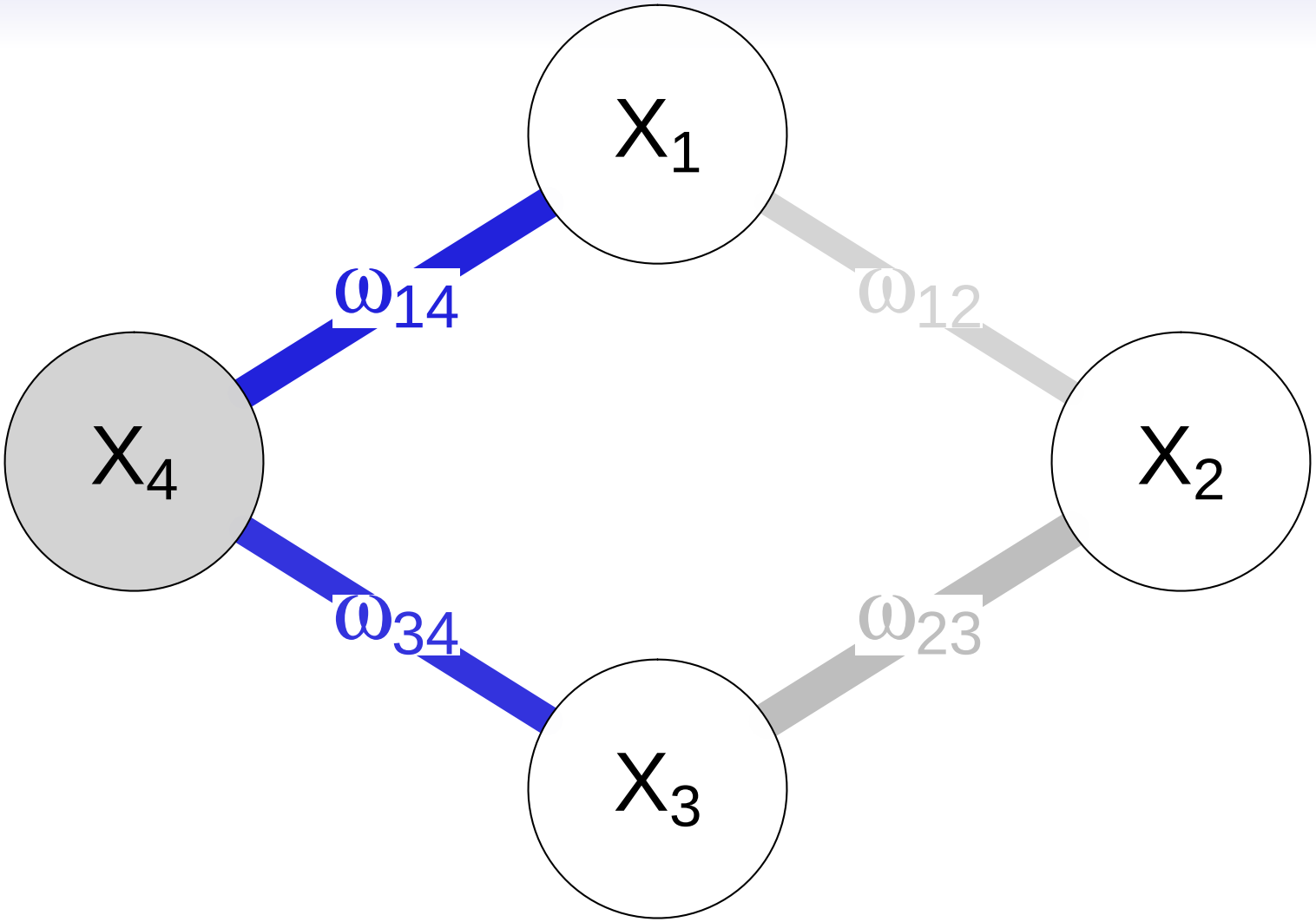
$$\Pr\left(X_1 = 1\right) \propto \exp\left(\tau_1 + \omega_{12}x_2 + \omega_{14}x_4\right)$$



$$\Pr(X_2 = 1) \propto \exp(\tau_2 + \omega_{12} x_1 + \omega_{23} x_3)$$



$$\Pr(X_3 = 1) \propto \exp(\tau_3 + \omega_{23} x_2 + \omega_{34} x_4)$$



$$\Pr(X_4 = 1) \propto \exp(\tau_4 + \omega_{14} x_1 + \omega_{34} x_3)$$

Estimating an Ising model

Multivariate estimation options:

- Optimize the *pseudolikelihood* $\prod_i \Pr\left(X_i = X_i \mid \boldsymbol{X}_{-(i)} = \boldsymbol{x}_{-(i)}\right)$
- Phrase the model as a loglinear model and use ML estimation

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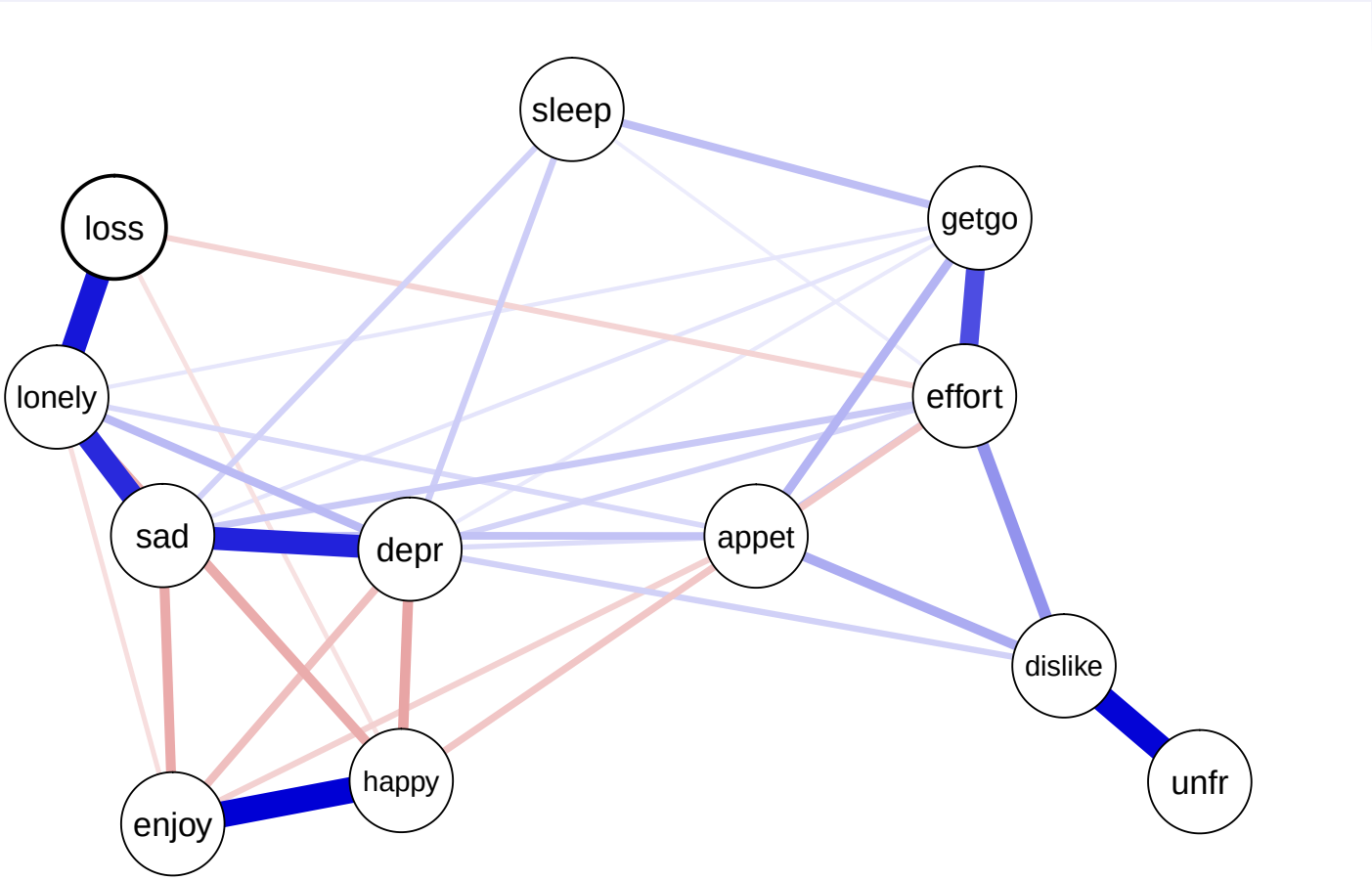
Methods for choosing which edges to include:

- Model selection
- Significance thresholding
- LASSO regularization
  - van Borkulo, C. D., Borsboom, D., Epskamp, S., Blanken, T. F., Boschloo, L., Schoevers, R. A., & Waldorp, L. J. (2014). A new method for constructing networks from binary data. *Scientific reports, 4* (5918).



Fried, E. I., Bockting, C., Arjadi, R., Borsboom, D., Amshoff, M., Cramer, A. O., Epskamp, S., Tuerlinckx, F., Carr, D., & Stroebe, M. (2015). From loss to loneliness: The relationship between bereavement and depressive symptoms. *Journal of abnormal psychology, 124*(2), 256-265.

The Ising Model & Psychometrics

- The Ising model can be shown to be mathematically equivalent to a multidimensional IRT model.
  - e.g., Marsman, M., Borsboom, D., Kruis, J., Epskamp, S., van Bork, R., Waldorp, L. J., van der Maas, H. L. J. & **Maris, G. K. J.** (2018). An introduction to Network Psychometrics: Relating Ising network models to item response theory models. *Multivariate Behavioral Research, 53(1)*, 15-35.
- Rank-1 clusters in the network correspond to latent variables in the MIRT model
- This equivalence allows for re-interpretation of MIRT models and network models alike

Adding (arbitrary) variance potential function $\ln \phi_{ii}(x_i) = \omega_{ii} x_i^2$, we can write the Ising model as:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \exp\left(\boldsymbol{\tau}^\top \boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{\Omega} \boldsymbol{x}\right)$$

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Now, reparameterize $\tau_i = -\delta_i$ and $-2\sqrt{\frac{\lambda_j}{2}} q_{ij} = \alpha_{ij}$ to obtain:

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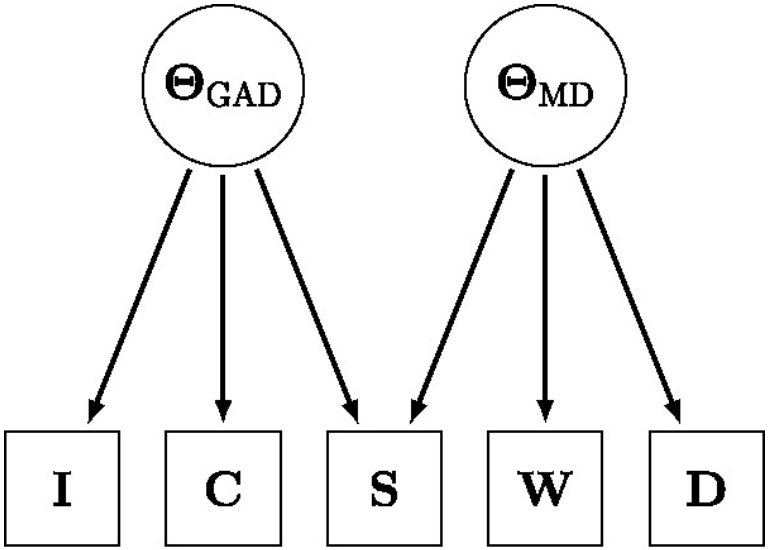
Now, reparameterize $\tau_i = -\delta_i$ and $-2\sqrt{\frac{\lambda_j}{2}} q_{ij} = \alpha_{ij}$ to obtain:

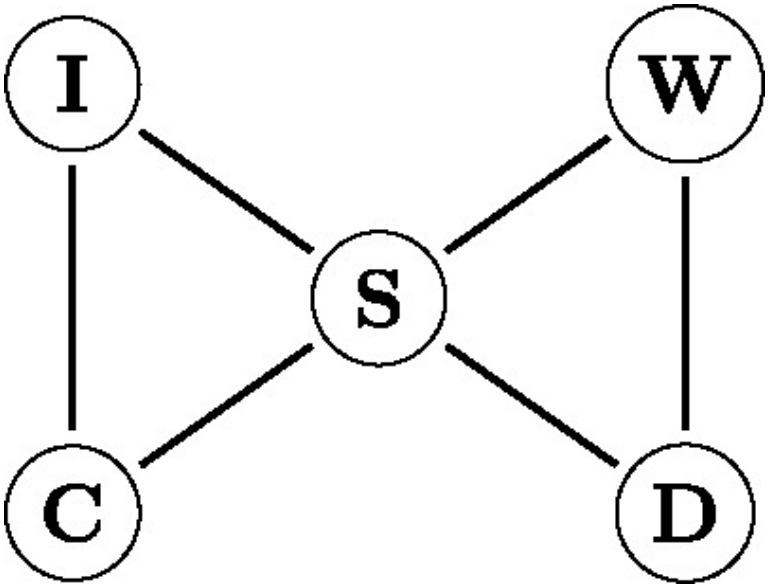
$$\Pr(\boldsymbol{X} = \boldsymbol{x}) = \int_{-\infty}^{\infty} f(\boldsymbol{\theta}) \Pr(\boldsymbol{X} = \boldsymbol{x} \mid \boldsymbol{\Theta} = \boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{\theta}$$

With:

$$\Pr(X_i = x_i \mid \boldsymbol{\Theta} = \boldsymbol{\theta}) = \frac{\exp\left(x_i \left(\boldsymbol{\alpha}_i^\top \boldsymbol{\theta} - \delta_i\right)\right)}{\sum_{x_i} \exp\left(x_i \left(\boldsymbol{\alpha}_i^\top \boldsymbol{\theta} - \delta_i\right)\right)}$$

$$\boldsymbol{\Theta} \mid \boldsymbol{X} = \boldsymbol{x} \sim N\left(\pm\frac{1}{2}\boldsymbol{A}^\top \boldsymbol{x}, \sqrt{\frac{1}{2}}\boldsymbol{I}\right)$$





I: irritability, C: chronic worrying, W: weight problems, D: depressed mood, S: sleep problems. Example from Marsman, M., Borsboom, D., Kruis, J., Epskamp, S., van Bork, R., Waldorp, L. J., van der Maas, H. L. J. & Maris, G. K. J. (2018). An introduction to Network Psychometrics: Relating Ising network models to item response theory models. *Multivariate Behavioral Research, 53(1)*, 15-35.

Gaussian Graphical Model

Now assume $X_i \in \mathbb{R}\,\forall_i$ and the log of all potential functions are *linear*, and include variance potential function $\phi_{ii}(x_i)$:

$$\ln \phi_i(x_i) = \tau_i x_i$$
$$\ln \phi_{ij}(x_i, x_j) = -\kappa_{ij} x_i x_j$$
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We obtain:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \exp\left(\boldsymbol{\tau}^\top \boldsymbol{x} - \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{K} \boldsymbol{x}\right)$$

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We obtain:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \exp\left(\boldsymbol{\tau}^\top \boldsymbol{x} - \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{K}\boldsymbol{x}\right)$$

Now let $\boldsymbol{\Sigma} = \boldsymbol{K}^{-1}$ and $\boldsymbol{\mu} = \boldsymbol{\Sigma}\boldsymbol{\tau}$:

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) \propto \exp\left(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{x} - \boldsymbol{\mu})\right)$$

Which implies $\boldsymbol{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$!

The Gaussian Graphical Model (GGM)

The precision matrix $\boldsymbol{K} = \boldsymbol{\Sigma}^{-1}$ encodes a network structure! Furthermore, this matrix can be standardized to *partial correlation coefficients*:

$$\mathrm{Cor}\left(X_i, X_j \mid \boldsymbol{X}_{-(i,j)}\right) = -\frac{\kappa_{ij}}{\sqrt{\kappa_{ii}}\sqrt{\kappa_{jj}}} = \omega_{ij}$$

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Combining this information, we can form a psychometric model:

$$\boldsymbol{\Sigma} = \boldsymbol{\Delta}\left(\boldsymbol{I} - \boldsymbol{\Omega}\right)^{-1}\boldsymbol{\Delta}$$

- $\boldsymbol{\Delta}$ is a diagonal scaling matrix
- $\boldsymbol{\Omega}$ is a symmetrical matrix with 0 on the diagonal and *partial correlation coefficients* on offdiagonal elements

GGM estimation

- The GGM can be fitted similarly to SEM models, allowing for model search strategies

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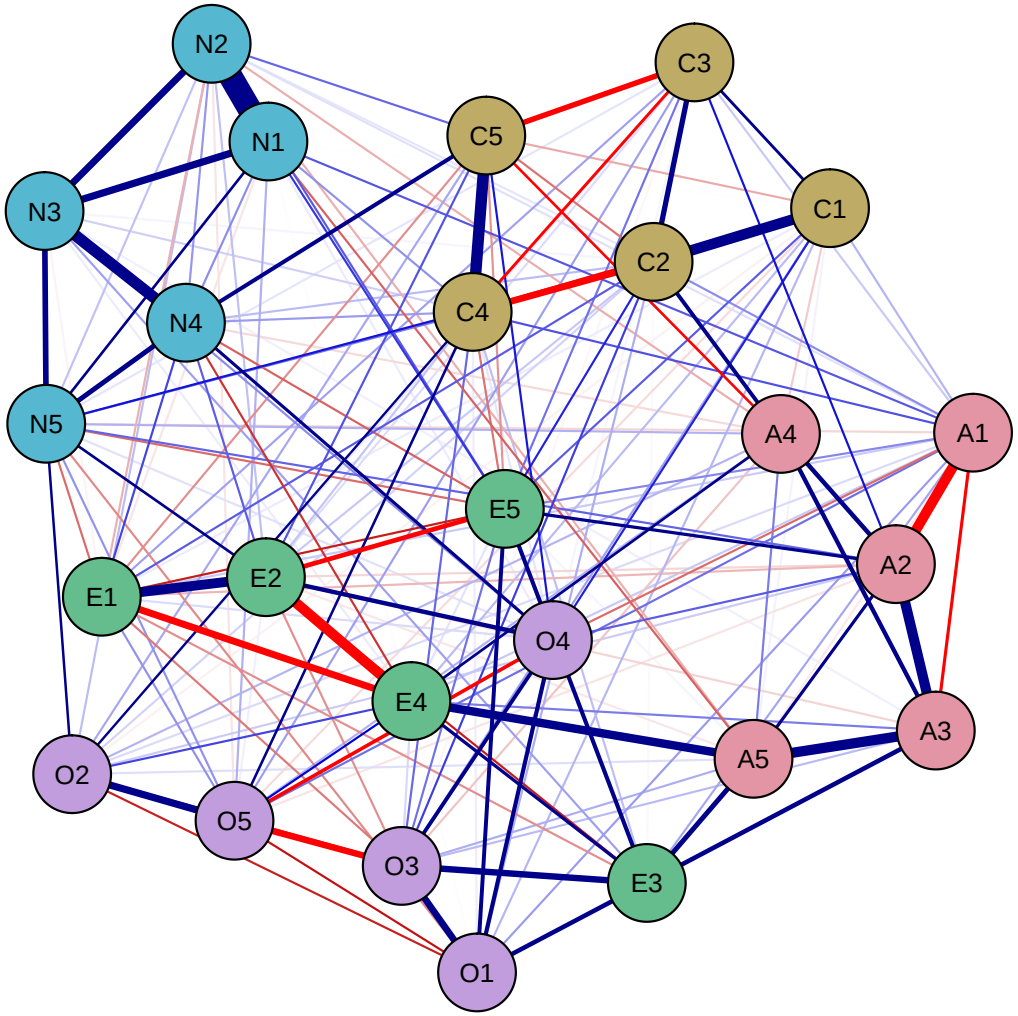
- Bayesian estimation methods are also promising



Agreeableness
- A1: Am indifferent to the feelings of others.
- A2: Inquire about others' well−being.
- A3: Know how to comfort others.
- A4: Love children.
- A5: Make people feel at ease.

Conscientiousness
- C1: Am exacting in my work.
- C2: Continue until everything is perfect.
- C3: Do things according to a plan.
- C4: Do things in a half−way manner.
- C5: Waste my time.

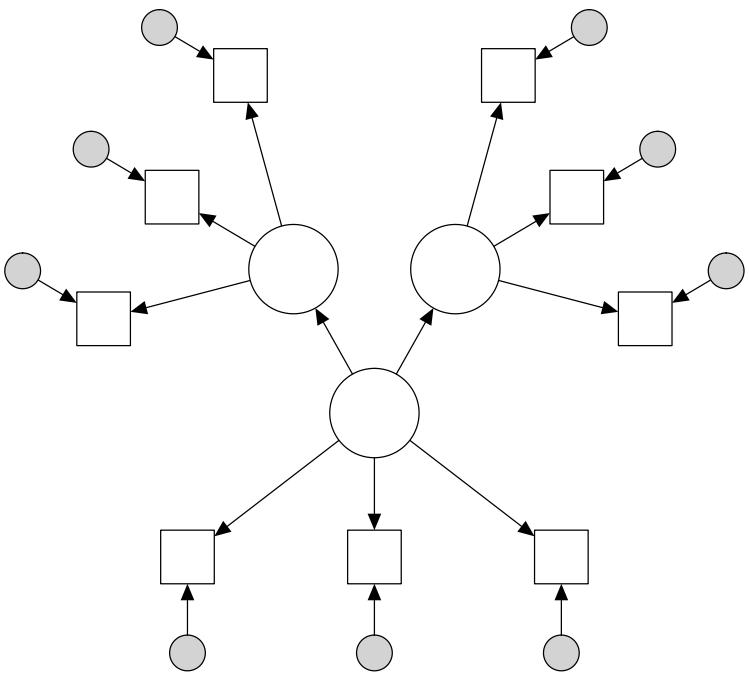
Extraversion
- E1: Don't talk a lot.
- E2: Find it difficult to approach others.
- E3: Know how to captivate people.
- E4: Make friends easily.
- E5: Take charge.

Neuroticism
- N1: Get angry easily.
- N2: Get irritated easily.
- N3: Have frequent mood swings.
- N4: Often feel blue.
- N5: Panic easily.

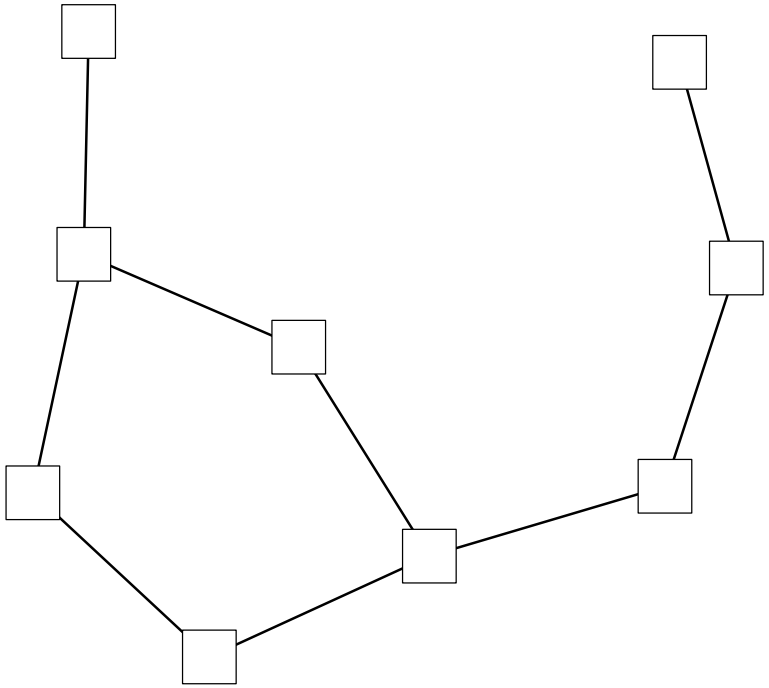
Openness
- O1: Am full of ideas.
- O2: Avoid difficult reading material.
- O3: Carry the conversation to a higher level.
- O4: Spend time reflecting on things.
- O5: Will not probe deeply into a subject.

SEM and GGM

Structural Equation Modeling

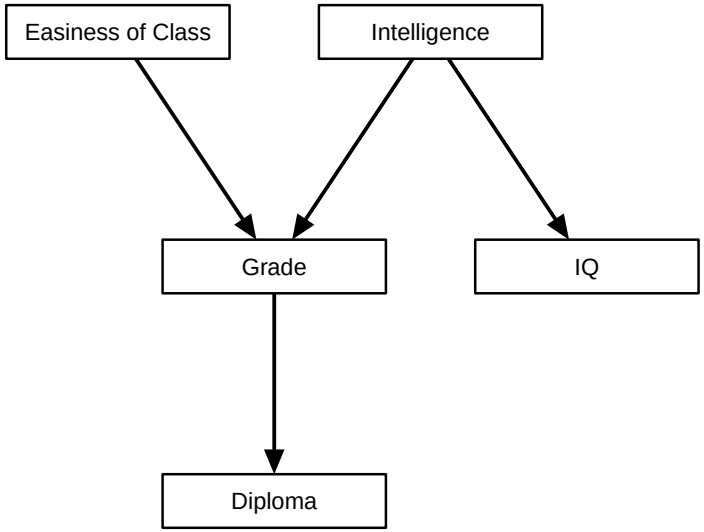


Gaussian Graphical Model

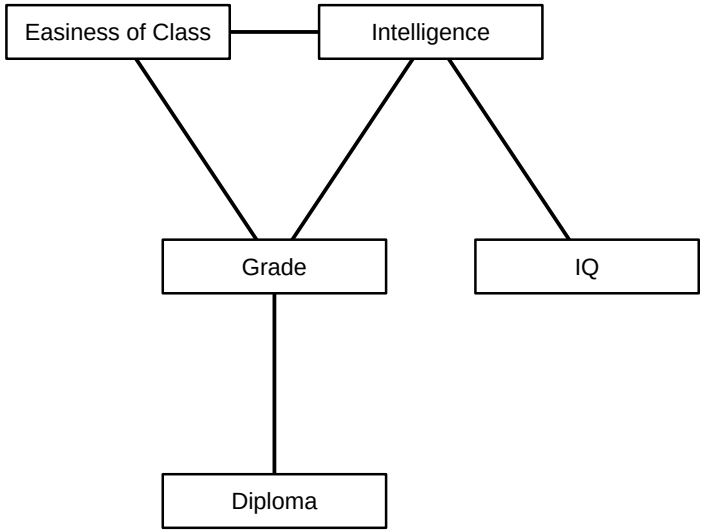


- Both models imply a variance-covariance matrix $\boldsymbol{\Sigma}$, aimed to closely resemble the sample variance-covariance matrix $\boldsymbol{S}$ with positive degrees of freedom.

Causal model



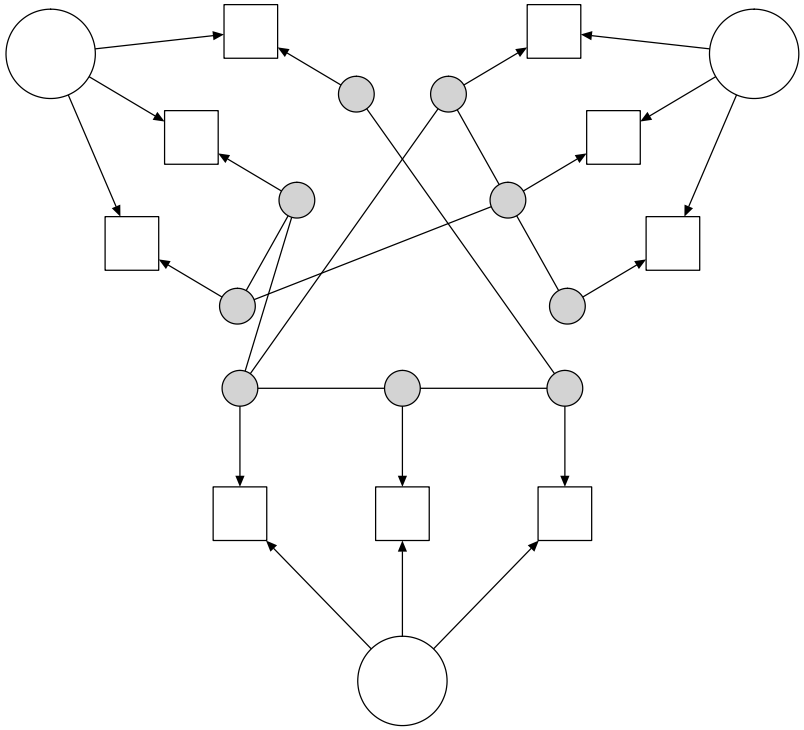
Markov Random Field



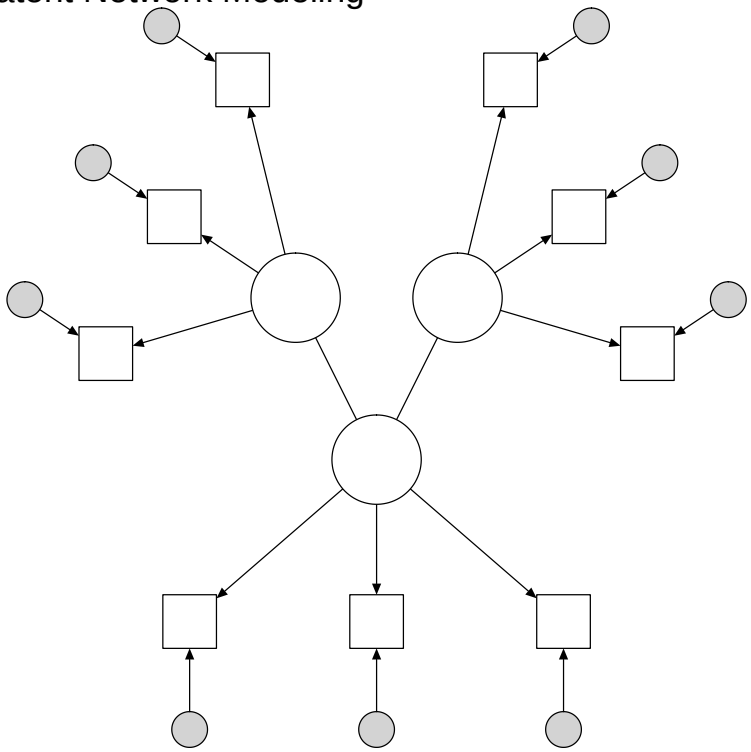
- MRF is uniquely identified and well parameterized
- MRF allows for exploratory hypothesis-generating insight in possible causal structures

Network models and SEM can be combined:

Residual Network Modeling



Latent Network Modeling



Epskamp, S., Rhemtulla, M., & Borsboom, D. (2017). Generalized network pschometrics: Combining network and latent variable models. *Psychometrika, 82*(4), 904-927.

Time-series

Graphical Vector Auto-regression (VAR)

$$\boldsymbol{X}_t \mid \boldsymbol{x}_{t-1} \sim N\left(\boldsymbol{B}\boldsymbol{x}_{t-1}, \boldsymbol{\Theta}\right)$$

- Variables assumed centered
- $\boldsymbol{B}$ encodes the *temporal network*
  - Temporal prediction
- $\boldsymbol{\Theta}^{-1}$ encodes the *contemporaneous network*
  - GGM
- Graphical VAR model
  - Wild, B., Eichler, M., Friederich, H. C., Hartmann, M., Zipfel, S., & Herzog, W. (2010). A graphical vector autoregressive modelling approach to the analysis of electronic diary data. BMC medical research methodology, 10(1), 28.

Estimation

- Multivariate multiple regression on lagged variables
- Edge selection via significance thresholding, model selection (LNM) or LASSO regularization
- Multi-level estimation allows for estimation on $N > 1$ datasets, but is more complicated
  - Two-step multi-level VAR using *mlVAR* R package
  - Bayesian estimation using *Mplus 8*

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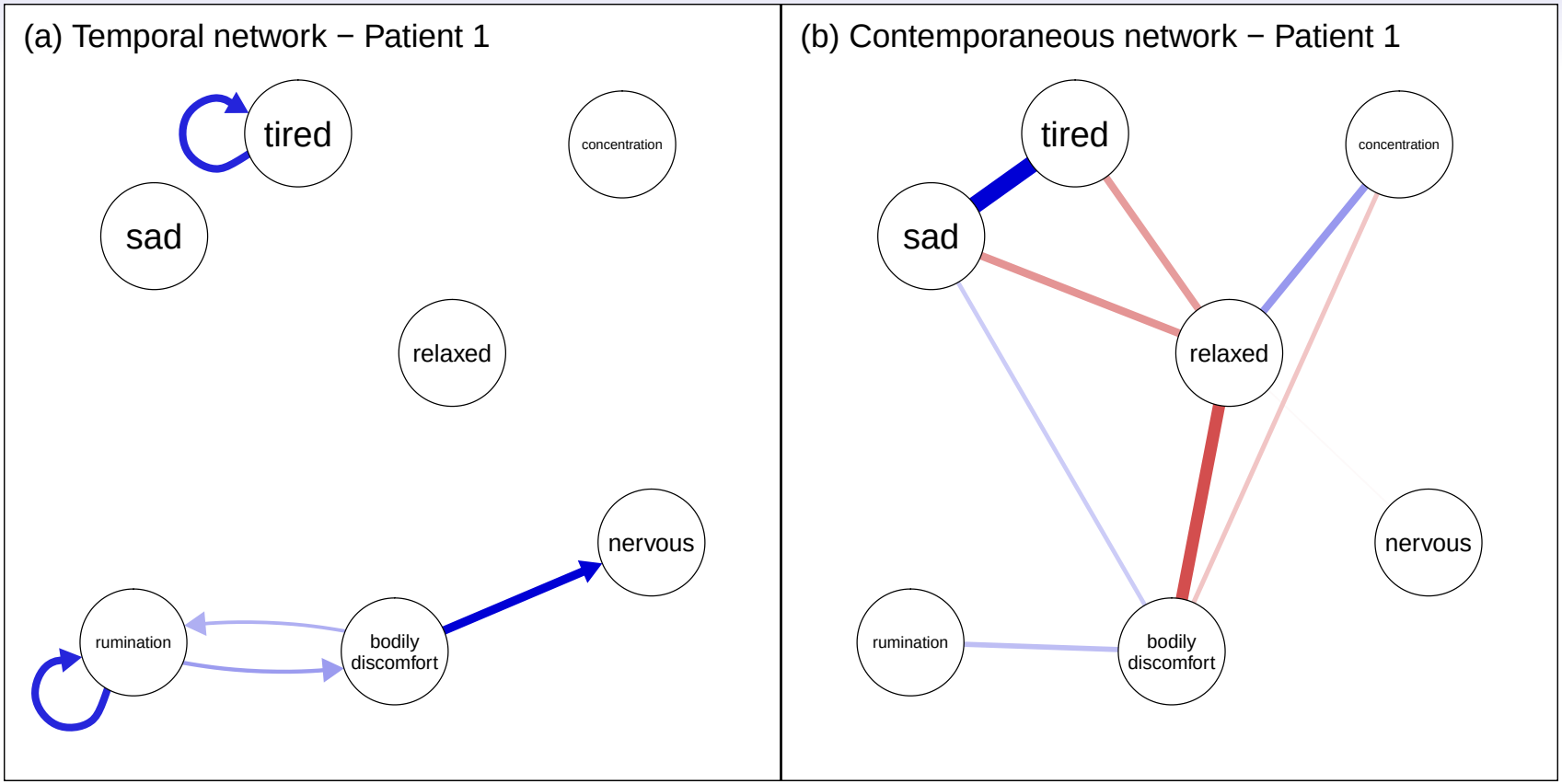
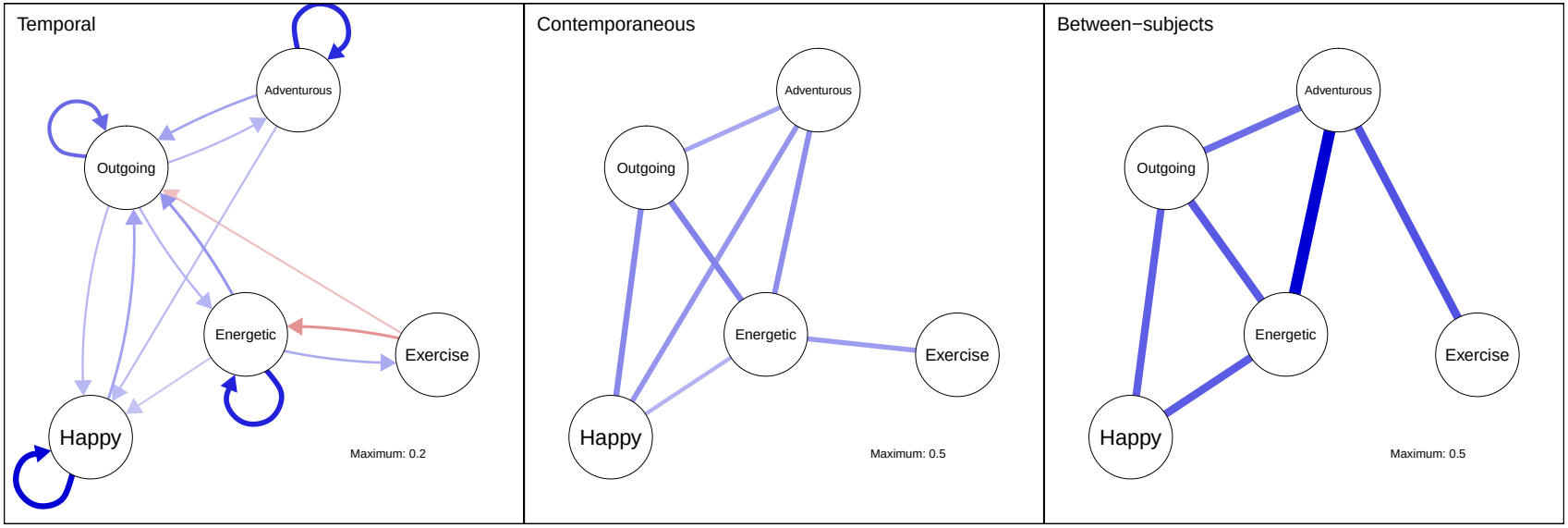
Important assumptions

- Stationarity
  - Means
  - Network structure
- Equidistant measurements



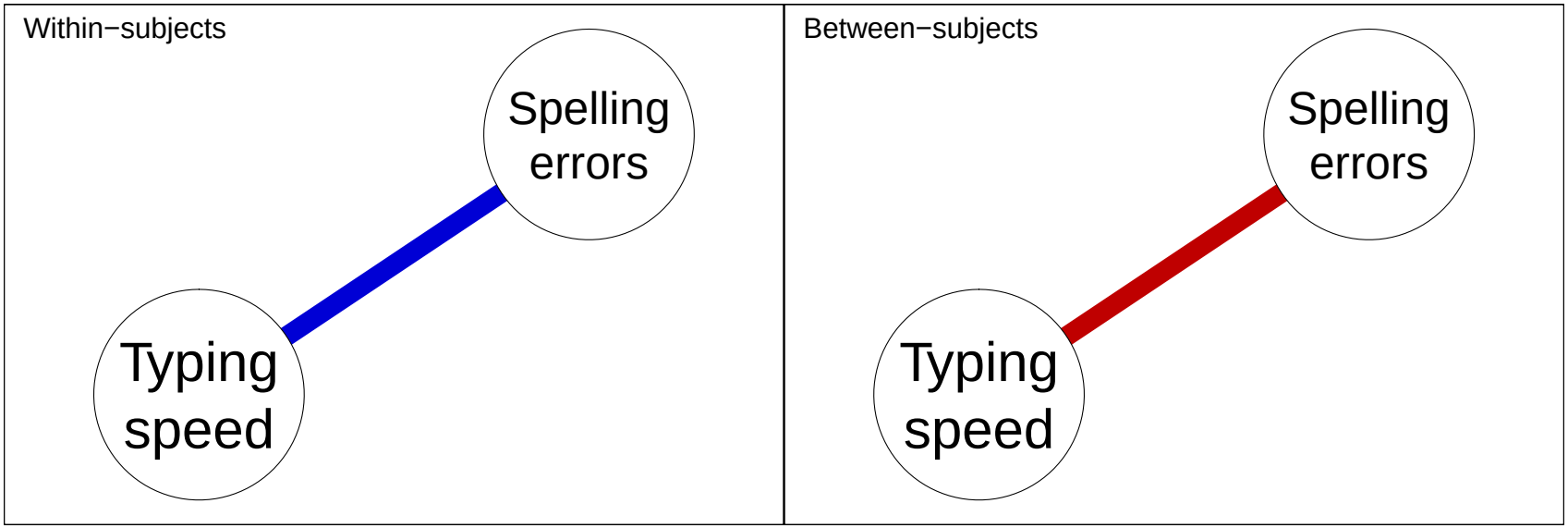
- Contemporaneous network: Partial correlations given $t-1$
- Temporal network: regression coefficients between $t-1$ and t

Epskamp, S., van Borkulo, C. D., van der Veen, D. C., Servaas, M. N., Isvoranu, A. M., Riese, H., & Cramer, A. O. (2018). Personalized network modeling in psychopathology: The importance of contemporaneous and temporal connections. *Clinical Psychological Science, 6*(3), 416-427.

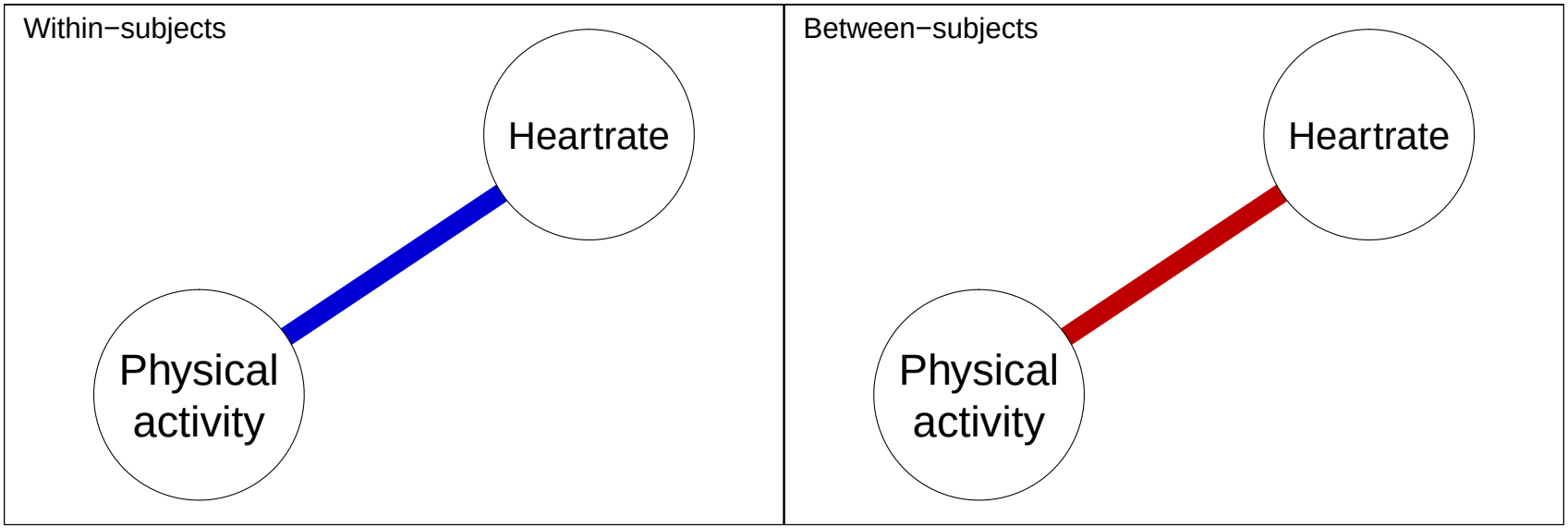


- Contemporaneous network: Partial correlations given $t-1$
- Temporal network: regression coefficients between $t-1$ and t
- Between-subjects network: Partial correlations between means

Epskamp, S., Waldorp, L. J., Mttus, R., & Borsboom, D. (2016). The Gaussian Graphical Model in Cross-sectional and Time-series Data. *Multivariate Behavioral Research.*



Example based on Hamaker, E. L. (2012). Why Researchers Should Think 'Within-Person': A Paradigmatic Rationale. *Handbook of Research Methods for Studying Daily Life*. The Guilford Press New York, NY, 43–61.

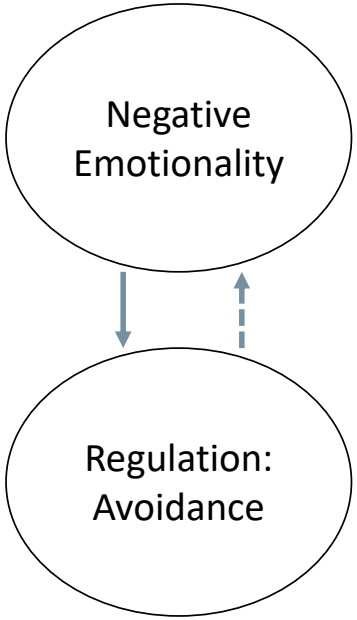


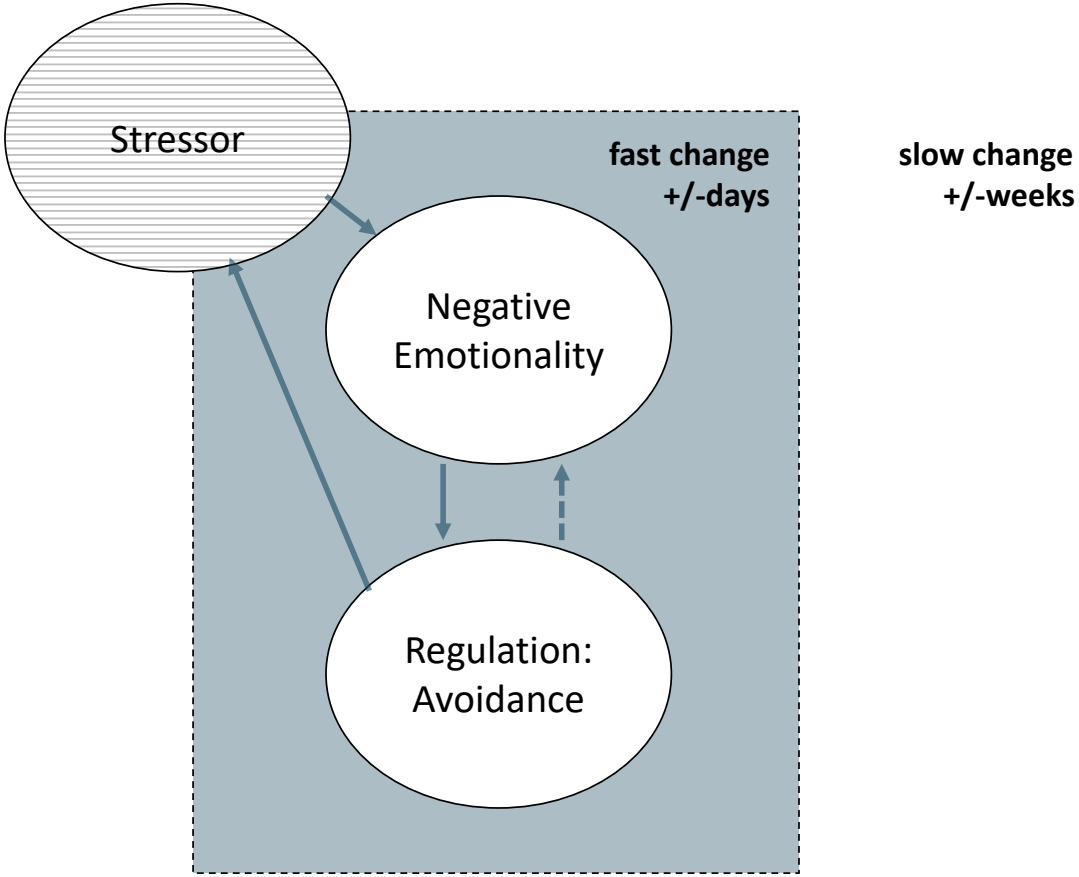
Example provided by Ellen Hamaker and based on Hoffman, L. (2015). *Longitudinal analysis: Modeling within-person fluctuation and change*. New York, NY, USA: Routledge.

Theory Formation

- Slides by Julian Burger
- Based on master-thesis project at the Amsterdam Institute for Advanced Studies
- Prospected PhD student 2019 - 2023
- Work together with Rick Quax and Don Robinaugh

UNIVERSITY OF AMSTERDAM
Institute for Advanced Study


Negative Emotionality
Regulation: Avoidance


Stressor
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slow change
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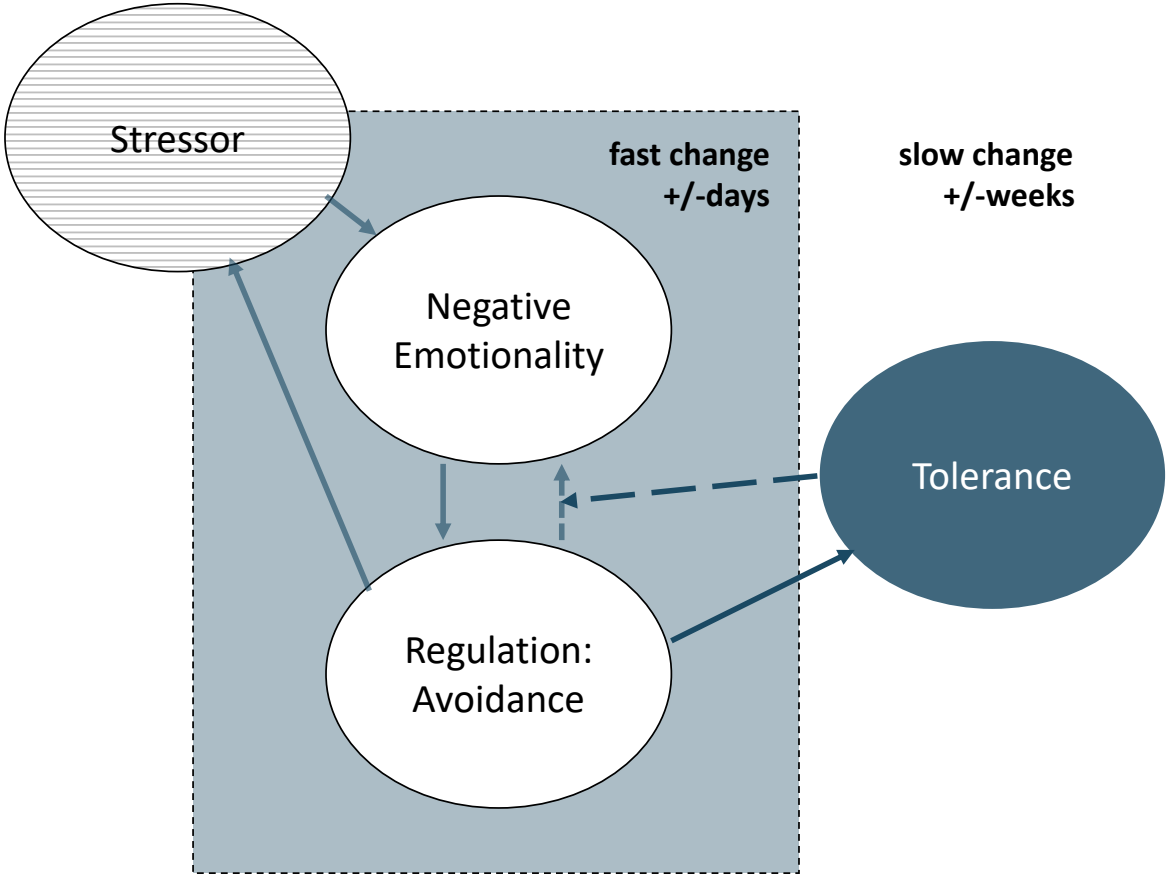
Negative
Emotionality
Regulation:
Avoidance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Regulation:
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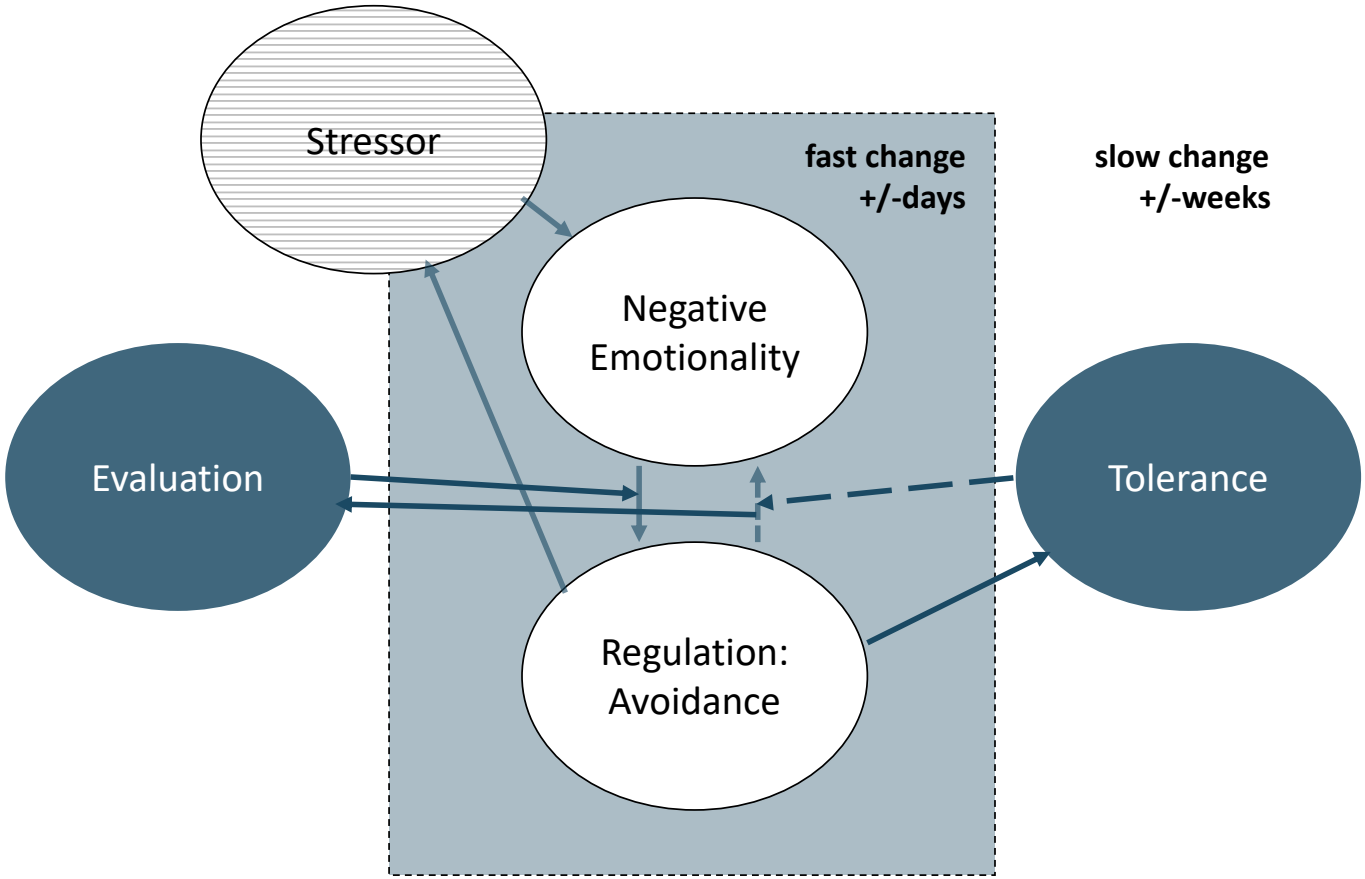
Stressor
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Emotionality
Evaluation
Tolerance
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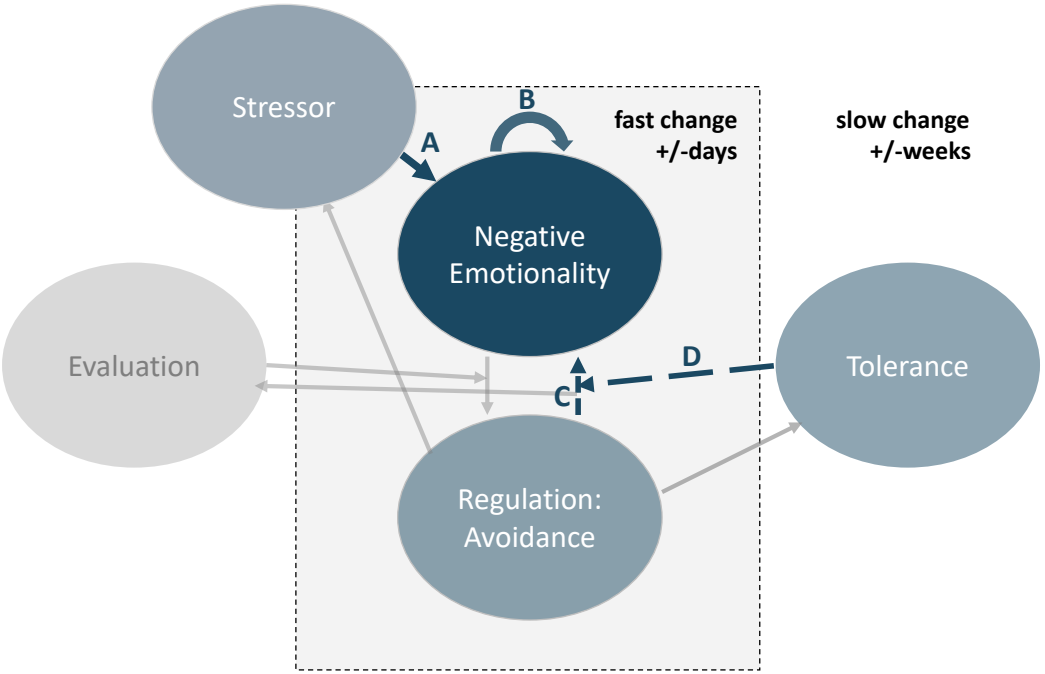
Negative Emotionality:

$$\frac{dNE}{dt} = \big((A * Str.) - B - (C - D * Tol.) * Av.\big) * NE * (1 - NE)$$



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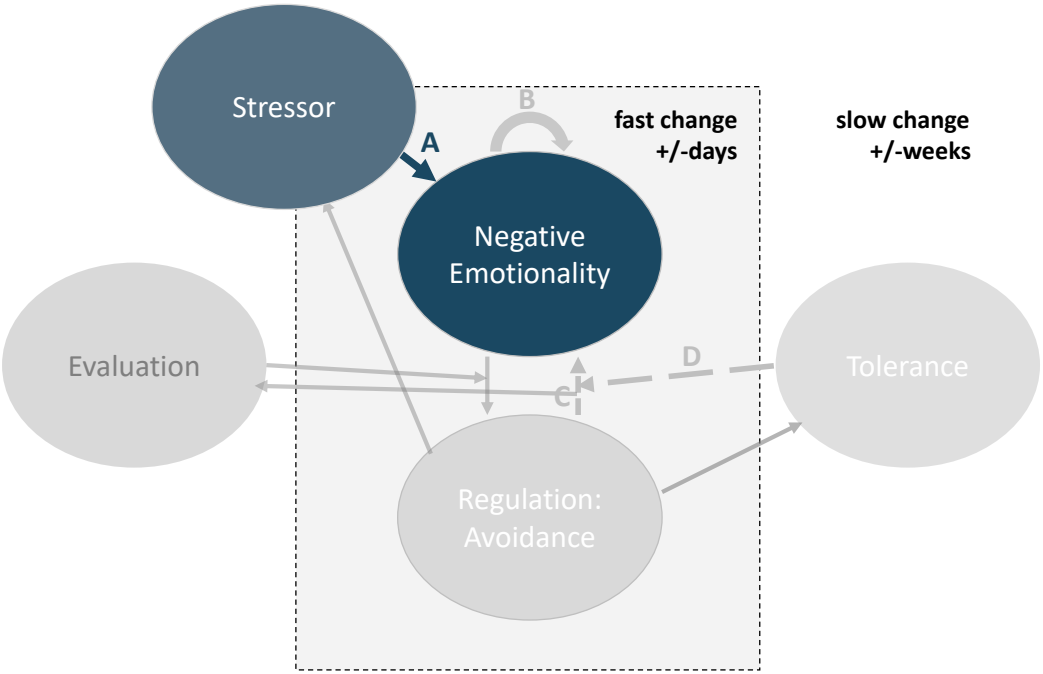

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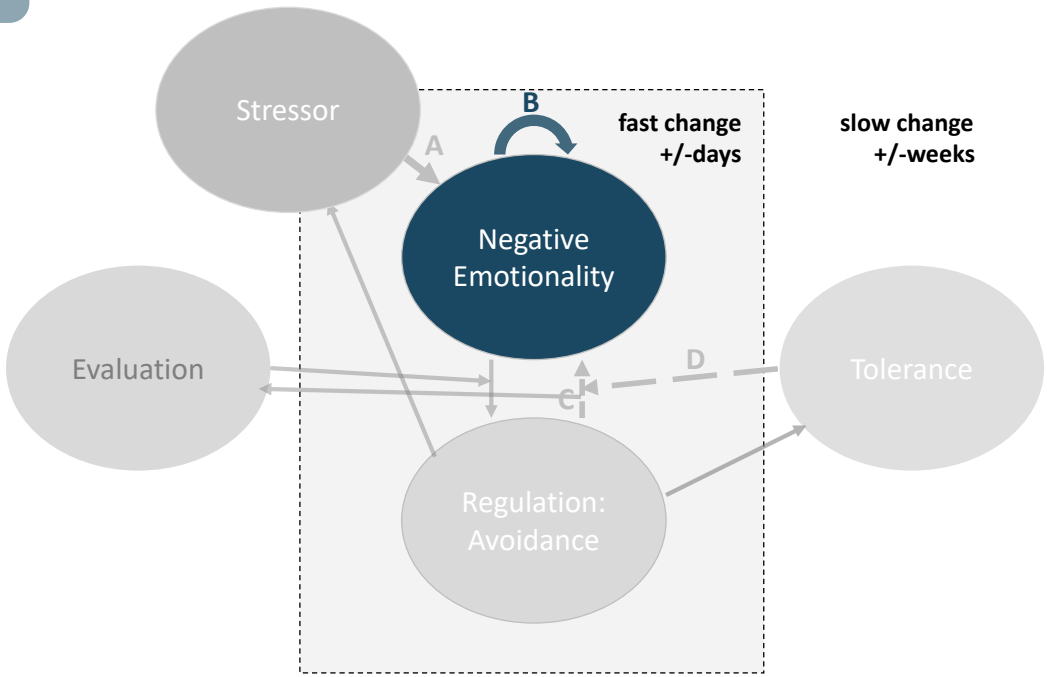

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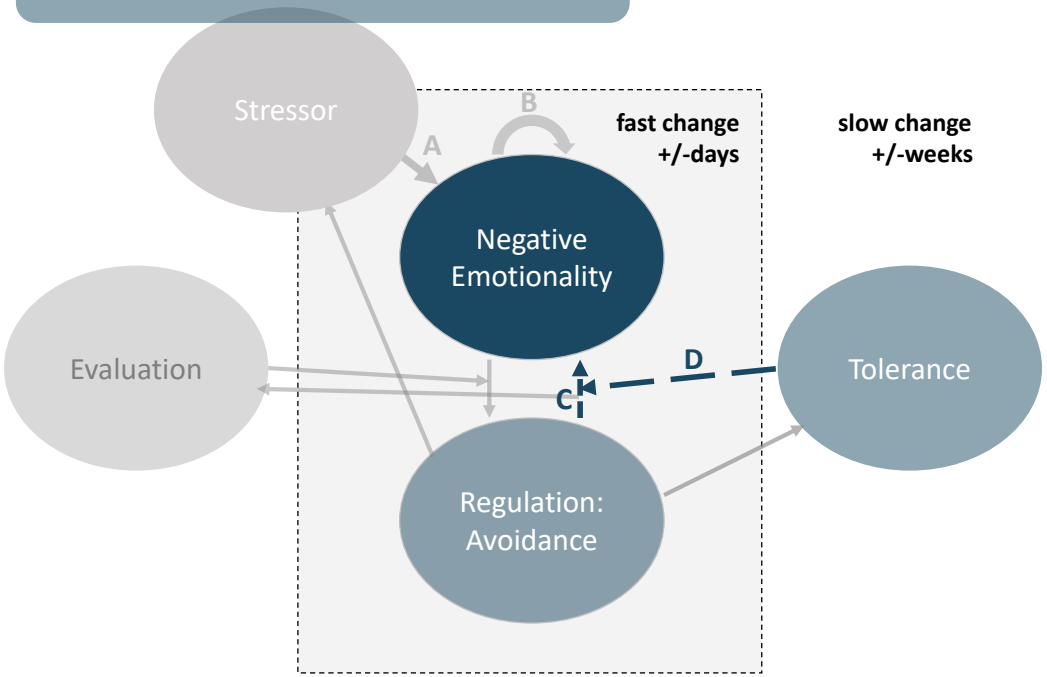

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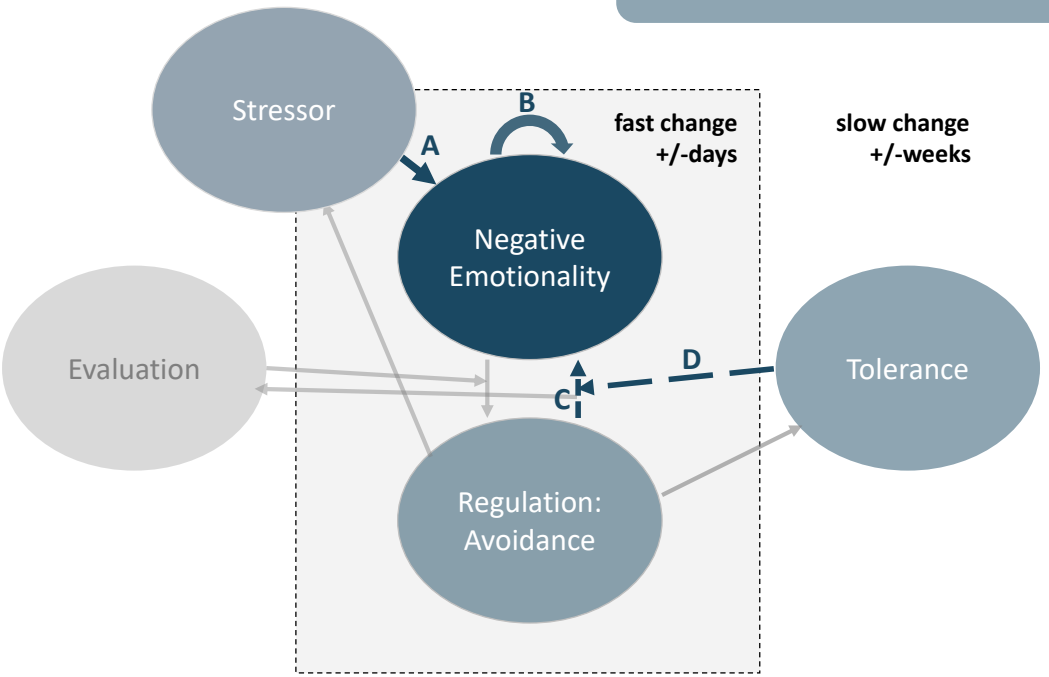

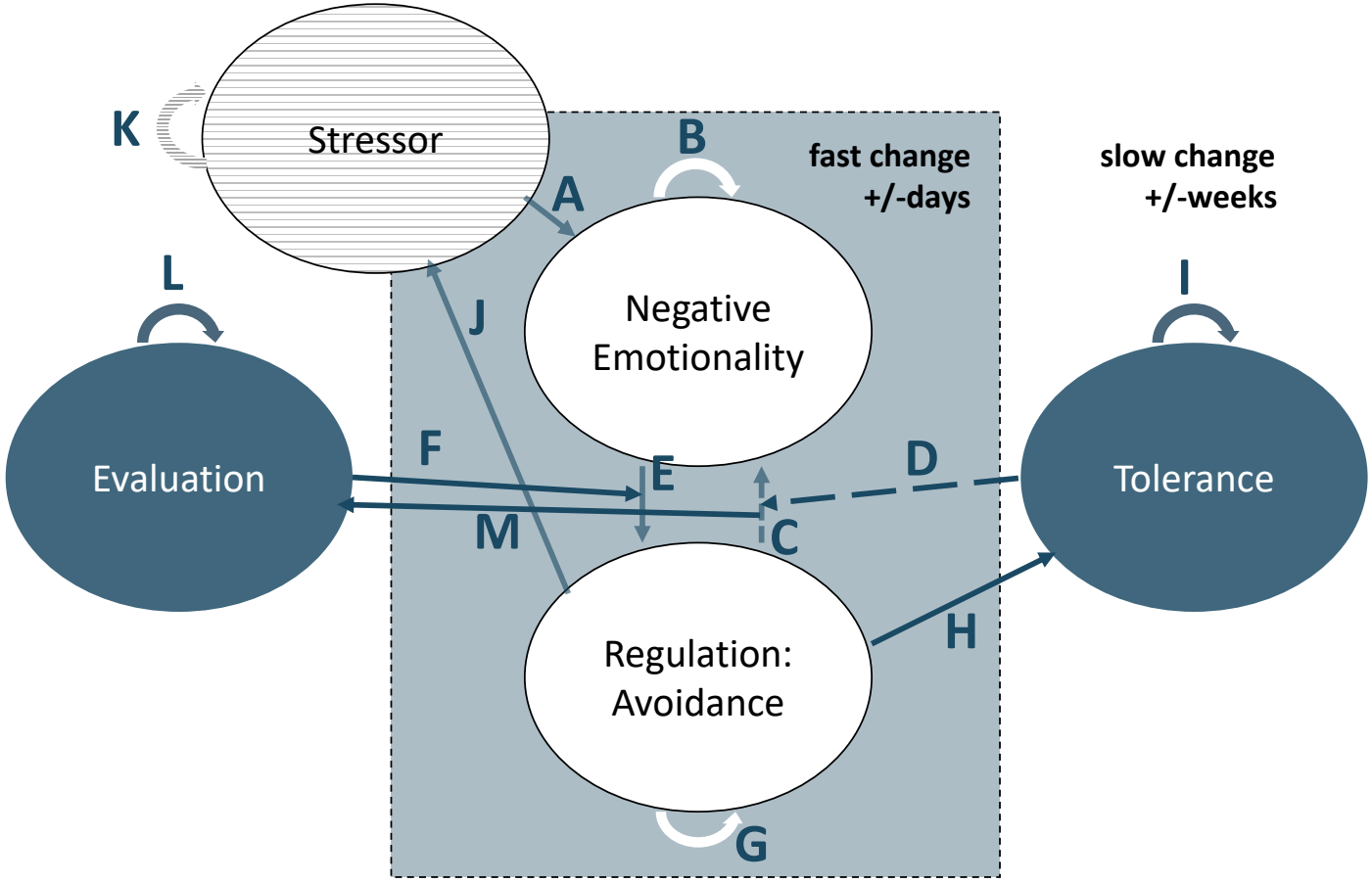
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H
Regulation:
Avoidance
G

Fast Change Equations

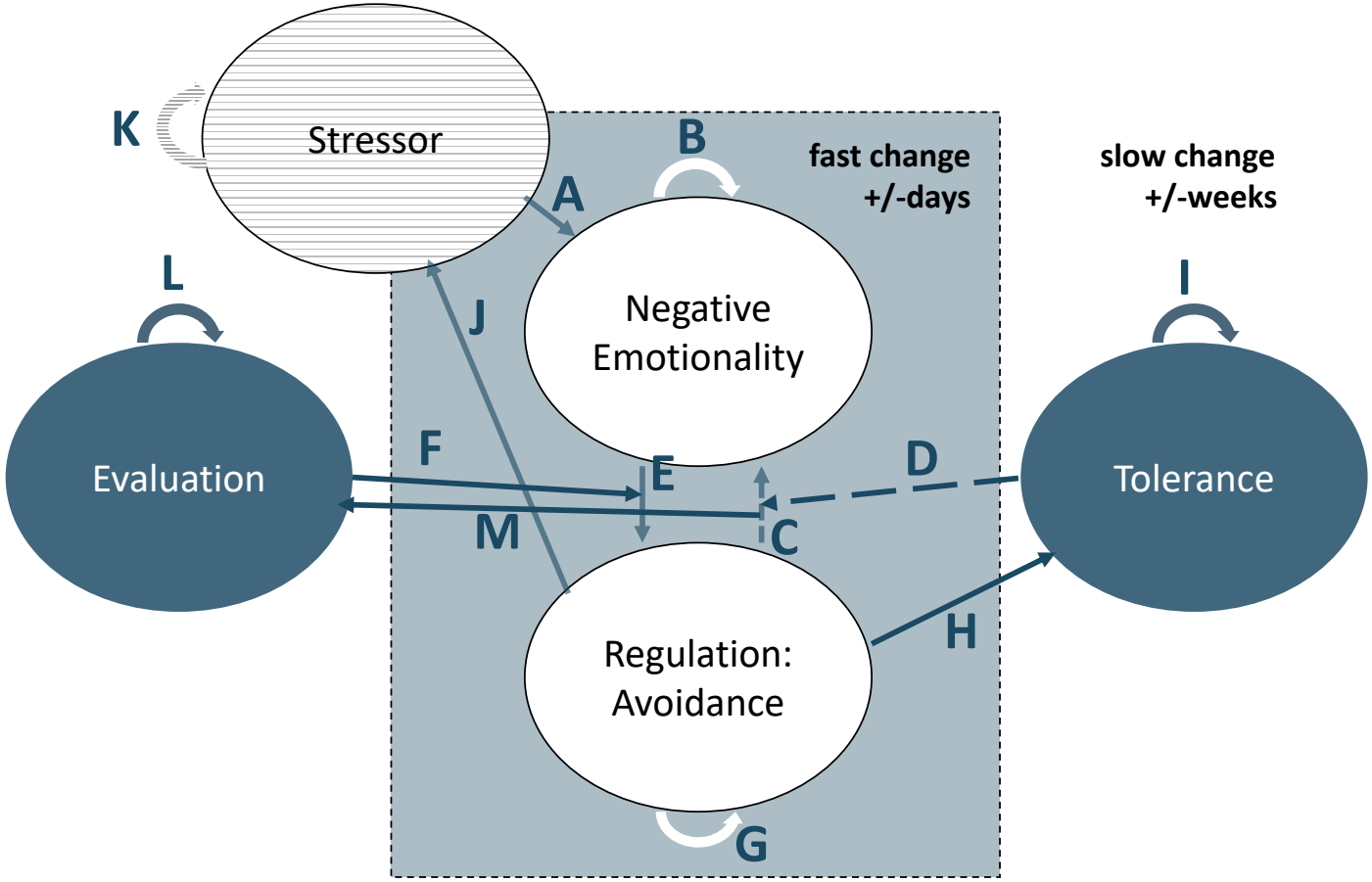
Negative Emotionality:

$$\frac{dNE}{dt} = \big((A * Str.) - B - (C - D * Tol.) * Av.\big) * NE * (1 - NE)$$

Regulation (Avoidance):

$$\frac{dAv.}{dt} = \big((E + F * Ev.) * NE - G * Av.\big) * Av. * (1 - Av.)$$

Scaling: 0 - 1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H
Regulation:
Avoidance
G

Slow Change Equations

Tolerance:

$$\frac{dTol.}{dt} = (H * Av. * (1 - Tol.) - I * Tol.)/7$$

Stressor:

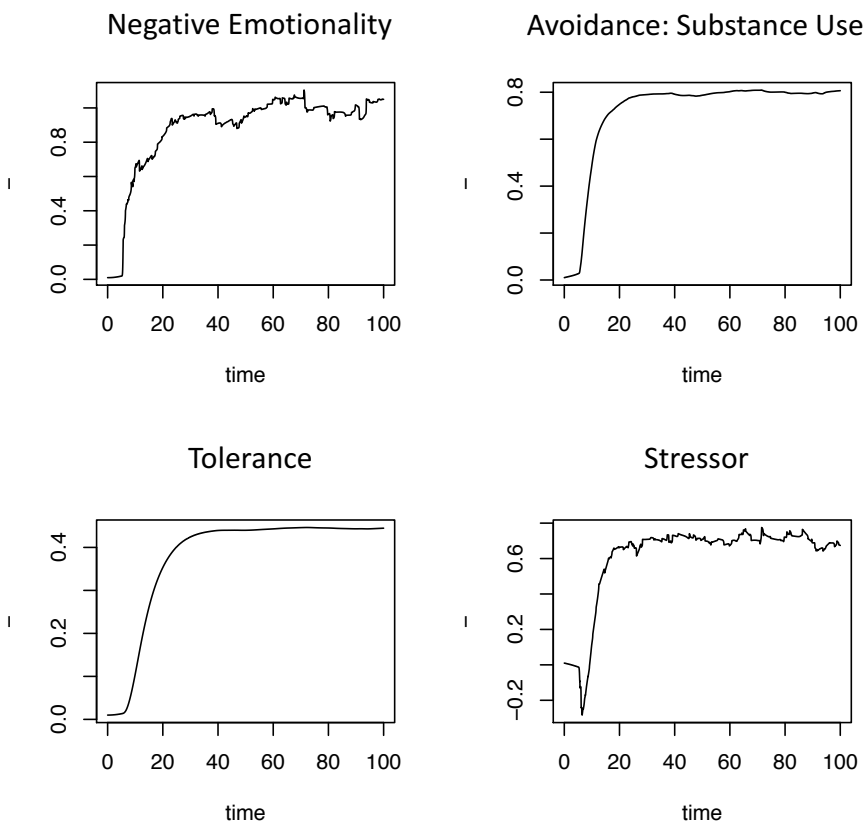
$$\frac{dStr.}{dt} = (J * Av. * (1 - Str.) - K * Str.)/7$$

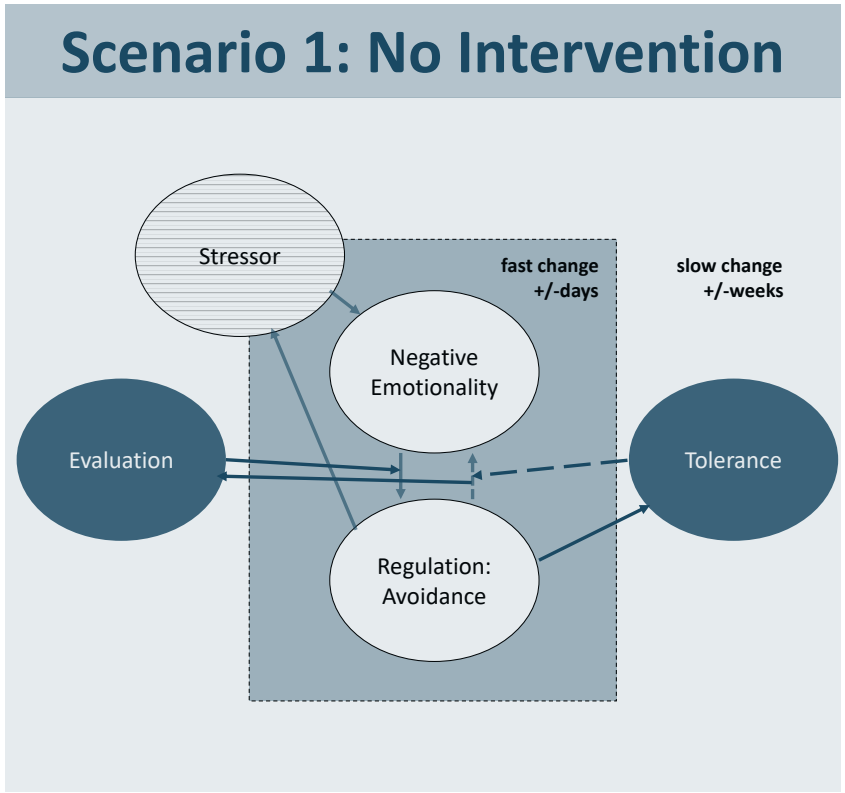
Evaluation:

$$\frac{dEv.}{dt} = (Ev. * (1 - Ev.) * C - L * Ev.)/7$$

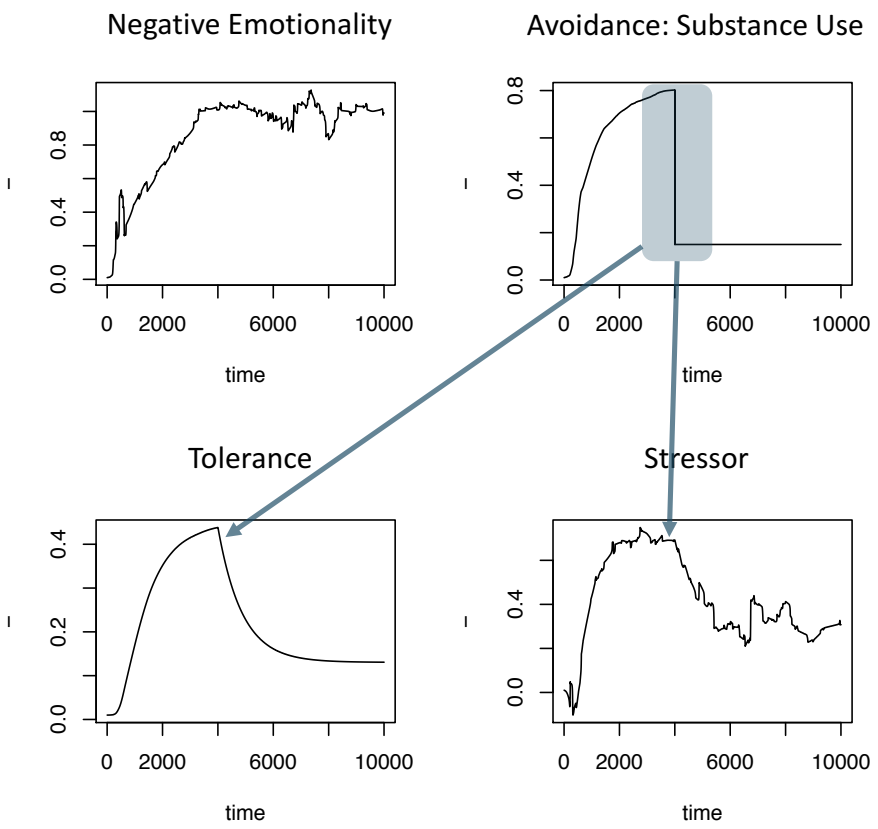
Delay in Change

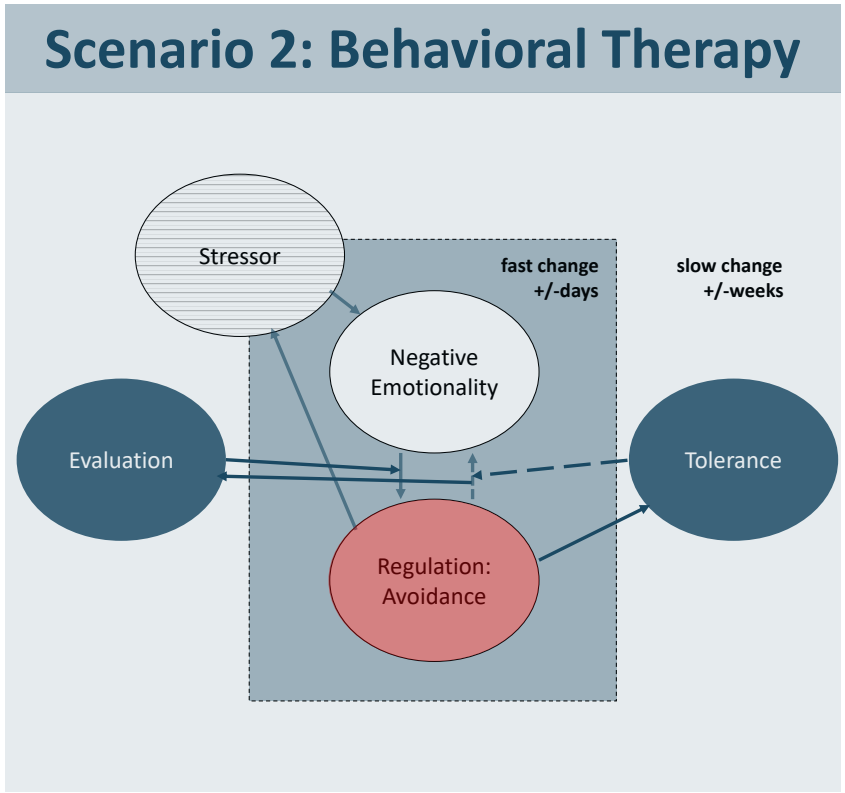
Incorporating random Mood Swings and Stressors



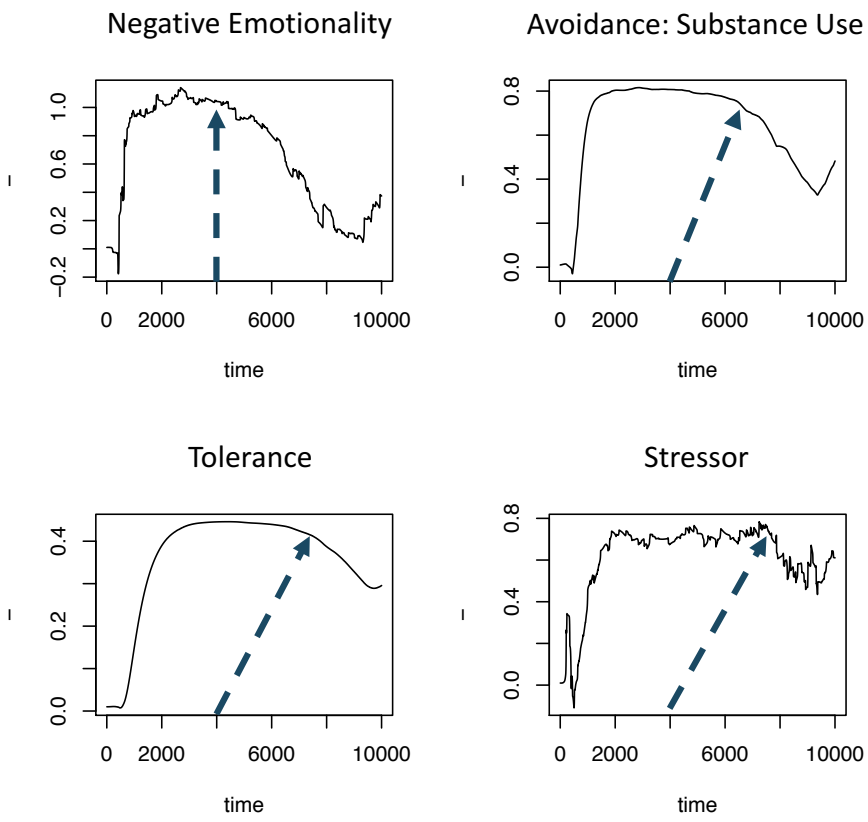


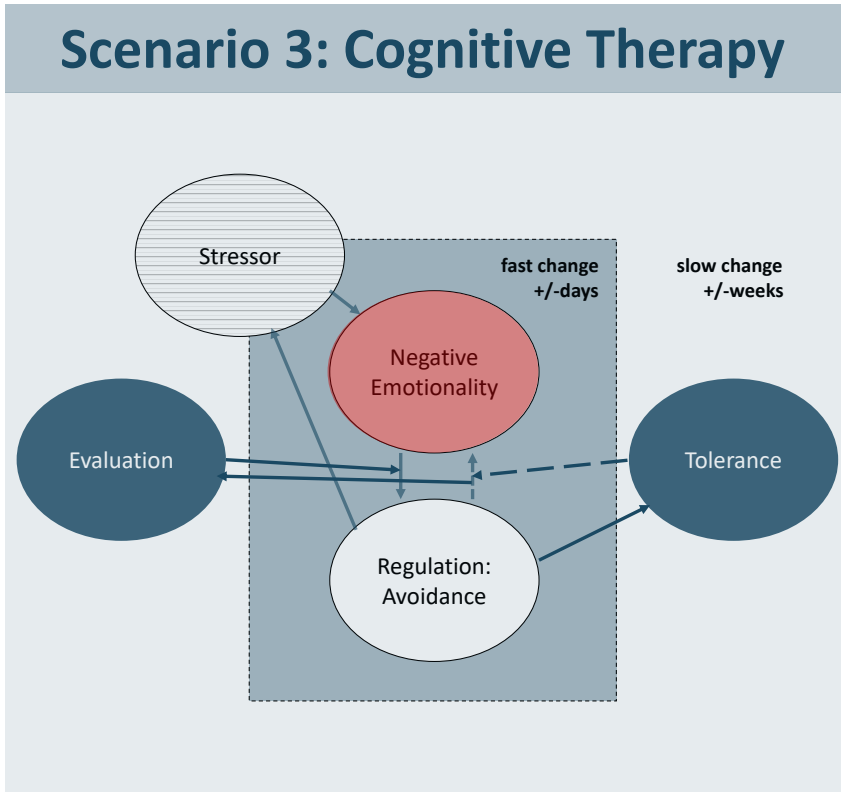
Intervention 1: Reduce Substance Use at t = 4000

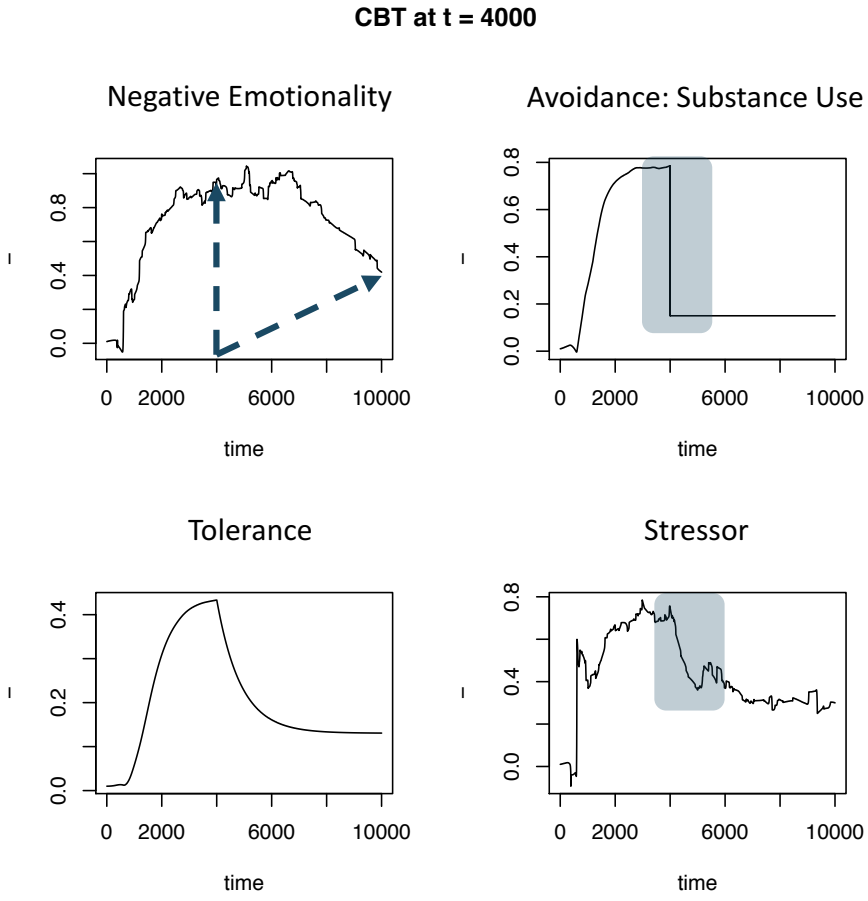


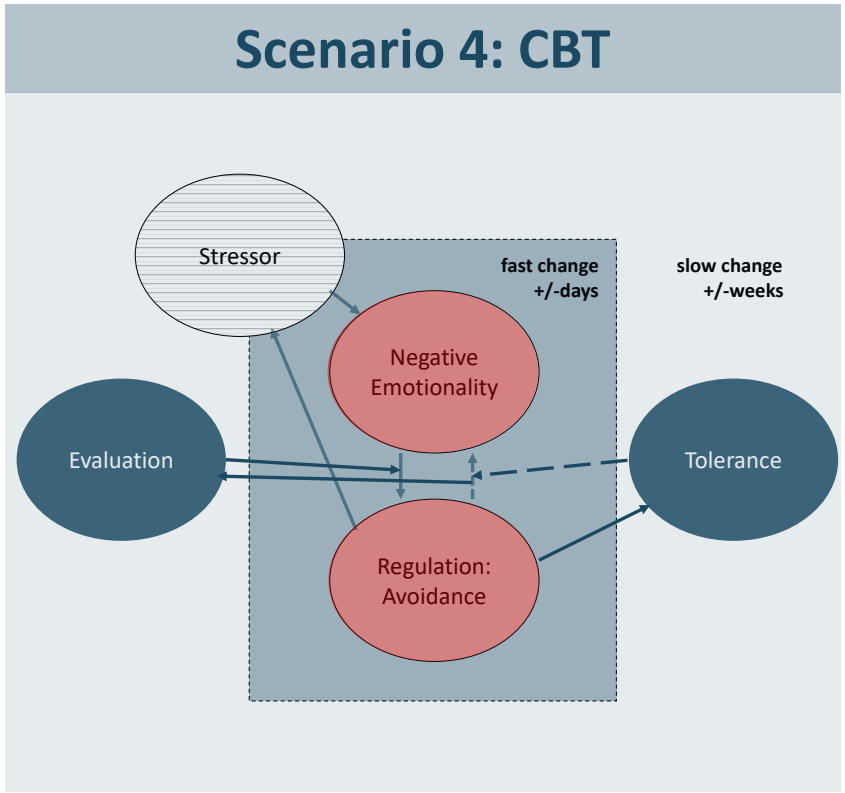


Intervention 2: Cognitive Therapy from t = 4000






CBT at t = 4000
Negative Emotionality
Avoidance: Substance Use
Tolerance
Stressor
time
0
2000
6000
10000
0.0
0.2
0.4
0.6
0.8


Scenario 4: CBT
Stressor
fast change
+/-days
slow change
+/-weeks
Negative
Emotionality
Evaluation
Tolerance
Regulation:
Avoidance

Conclusion

Current state of Network Psychometrics

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- Popular Facebook group!
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  - Build networks from the ground up, based on theory
- Dynamical systems modeling of psychological phenomena is promising

Thank you for your attention!

- Website: `sachaepskamp.com`
  - Slides at: `sachaepskamp.com/presentations`
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